PATTERN RECOGNITION FEATURE AND IMAGE PROCESSING THEORY ON THE BASIS OF STOCHASTIC GEOMETRY[1]

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Abstract: Application of stochastic geometry methods to pattern recognition is analysed. The paper is based on Trace-transformations of original images into images on the Möbius band. Based on the new geometric transformation, a new approach towards the construction of features, independent of images' motions or their linear transformations, is put forward. A prominent characteristics of the group of features under consideration is that we can represent each of them as a consecutive composition of three functionals. The paper considers the application of three-functional structure of recognition feature to image pre-processing. Feature can be invariant or sensitive to the group of all motions transformation and linear deformation of objects depending of functionals selection. Thus sensitive features are suitable to determine the parameters of translation. It is an important task for robotics.

1 INTRODUCTION

In the field of pattern recognition we traditionally distinguish feature construction and decision procedure. In literature on cybernetics a vast majority of works on the pattern recognition have been historically devoted to decision rules, there actually being no works on feature construction. There has been general agreement that it could be explained by the fact that the process of constructing features is empirical and dependent on the intuition of the recognition system designer.

The approach of stochastic geometry, developed in (Fedotov, 1990), allows us to bridge the gap and create processes to generate great many new features for image recognition, along with a constitutive theory of features. Such a prominent shift of stress from decision procedures to new recognition features gives the approach a strong resemblance to neuro-computing.

In (Fedotov, 1990), the author suggests using probabilities of geometrical events understood as the result of geometrical objects interaction (intersections, overlapping and so on), as image recognition features. Geometrical objects here are, on the one hand, complex scanning trajectories with random parameters (segments, lines, curves, figures, etc.), and on the other hand, fragments of an image being recognized. The structure of similar recognition systems and examples of particular technical implementations, are considered. Possible extensions of the fundamental recognition process on stochastic geometry are considered as well. One of the extensions deals with a complication of observing a random event (an intersection of a scan trace and an image), i.e. with the application of more complicated recognition features. The article presents the basics of a new theory to construct recognition features based on stochastic geometry and functional analysis joint application. The key element of this theory is new geometric Trace-transform concerned with scanning of image by complicated trajectories (Fedotov, 1990) (Fedotov et al., 1995).

Trace-transform can be both the base for the new class of recognition features – the triple ones – and the effective tool of image preprocessing.

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2 TRACE-TRANSFORM

Let F denote a finite image. Given scanning straight line l, g characterizing the location of l and the image as to each other, is to be computed according to a certain rule $\mathbf{T}$**:** $g = \mathbf{T}(l, F)$; map $\mathbf{T}$ is called a functional. The property required for us here is the computational independence of an object motion. Therefore the only requirement imposed on $\mathbf{T}$ is stated as follows. Let the image have been shifted and rotated, and F' be the new one. For the same shift and rotation, l will become a straight line l', thus remaining, "frozen" into the image. It is required that $\mathbf{T}(l, F) = \mathbf{T}(l', F')$. The equality is to hold for all straight lines and all acceptable images. We may call the property a complete invariance of functional $\mathbf{T}$. It should be noted that the concept of complete invariance extends pattern recognition capabilities substantially, for it is not necessarily be the number of intersections, intersection length, etc.

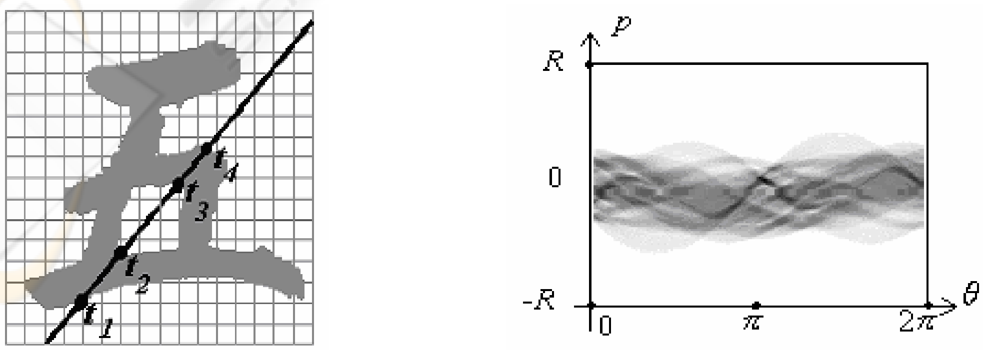
Functionals can be selected to describe finer properties of neighborhood, such as neighborhood morphology, or topological characteristics. For a full-color image of a variable brightness a great number of such functionals could be selected. Hence, the range of functionals and images to be processed widens considerably.

Just like in stochastic geometry, random value $g = \mathbf{T}(l, F)$ is defined, its distribution being independent of image shifts and rotations. Therefore, numeric characteristics of the random value may again serve as image features, which are to be established with the help of special engineering devices and systems. The limitation of the new family of features is that they originally lack an explicit geometrical meaning, and their differentiating capability is a priori unknown. However for pattern recognition, it proves not very important, experimental testing being decisive.

To understand that the generalization proposed in a certain aspect exhaust its own possibilities, we are going to state the theory of Trace-transform. Polar coordinates introduced to the plane, l is characterized by distance p from the origin to l, and by angle θ (up to 2π) of its directional vector: $l = \{(x, y) : x\cos\theta + y\sin\theta = p\}$, $l = l(\theta, p)$, where x, y are Cartesian coordinates on the plane. If we allow p to take negative values, too, then $l(\theta, p) = l(\theta + \pi, -p)$.

Thus, a set of all directed straight lines intersecting a circle of radius R with the center in the origin (the "retina"), is unambiguously parameterized by set $\Lambda = \{(\theta, p) : 0 \le \theta \le \pi, -R \le p \le R\}$, which provided parameters $(0, p)$ and $(\pi, -p)$ define the same straight line. The set of straight lines on the retina are clearly seen to be topologically nothing but a Möbius band. Thus, the set of numbers $\mathbf{T}(l(\theta, p), F)$, depending on a point on Möbius band Λ, is a certain image transform, which we may call a Trace-transform. If, for instance, a matrix represents a Trace-transform in numerical analysis, we may call it a Trace-matrix. If axis 0θ is directed horizontally, and axis $0p$ vertically, matrix element, indicated (i, j), i.e. value $\mathbf{T}(l(\theta_j, p_i), F)$, is in point θ_j, p_i. θ_j and p_i are here certain values of uniform discrete grids on the axes mentioned. Along the horizontal axis, matrix is 2π-periodic, its columns rotating within each interval of length π.

In addition, let us consider, that if l does not intersect the image, $\mathbf{T}(l, F)$ is a given number (say,



a) b)

Figure 1: Example of calculation a binary function $f(\theta, p, t)$ for given image a scanning line l and corresponding Trace-Transform

a) b)

Figure 2: Nonlinear filtration by means of Trace-transform

0), or another fixed element, if $\mathbf{T}$ is nonnumeric. In this case, a new image $\mathbf{T}(F)$ corresponds to the original image F, $(\mathbf{T}(l(\theta, p), F)$ may be treated as an image which characteristics at (θ, p) are its Trace-image.

Figure 1,a explains the computation of a Trace - transform. It shows how to obtain a binary function $f(\theta, p, t)$ of a real variable for a scanning line l. Function $f(\theta, p, \bullet)$ equals 1 within the interval (t_1, t_2) and (t_3, t_4). Within other precise it equals 0. Let $\mathbf{T}$ stand for a functional applied to the function, its independent variable being designated by t. Thus we get $g(\theta, p) = \mathbf{T}\, f(\theta, p, t)$. We call function g result of Trace–transform.

For a two–level image, such a Trace–transform could be obtained in case $\mathbf{T}\, f(\theta, p, \bullet)$ is the total of all the intervals from the domain of the function to be defined. For Figure 1,a it is the value of $t_2 - t_1$ and $t_4 - t_3$ segments' total. If we determine a similar $\mathbf{T}\, f(\theta, p, \bullet)$ for an aggregate of scanning lines intersecting the image of a image at various angles θ and various distances p, we can get its Trace–image shown in Figure 1,b.

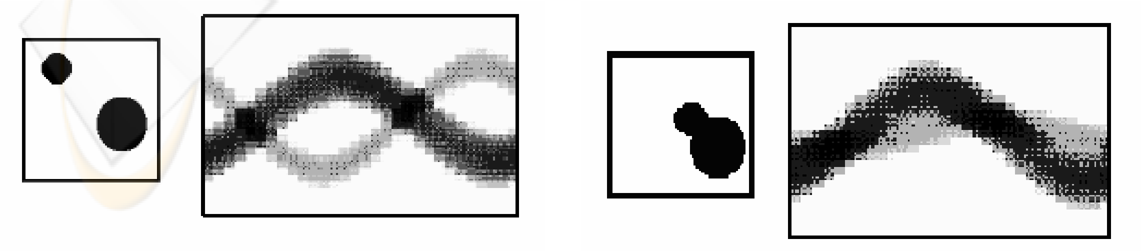
Let $\mathbf{T}\, f(\theta, p, t)$ be maximum interval within function $f(\theta, p, \bullet)$ domain. In Figure 1,a it is the value of $t_2 - t_1$ (max *G*). This functional get us another Trace-image.

Note that the famous Radon transform (Helgason , 1980) can be viewed as an example of a Trace–transform. Brightness integral value summed over all scan lines for all directions, is Radon transform. In case $\mathrm{T} f_{p,\theta} = \int_{-\infty}^{\infty} f_{p,\theta}(t)dt$ — brightness integral value $f(p, \theta)$ along a scan line with parameters (θ, p), the collection $\{\mathrm{T} f_{p,\theta}\}, p \in R, \theta \in [0, 2\pi]$ bears all information on the image.

In the context of two-level images, Radon transform will lie in summing up blackened elements along each scan line. Hence, choosing an appropriate functional, we can make Trace-transform realize Radon transform. This is a particular case, though. In other particular cases one can both make Trace–transform match other well-known geometric transforms — Fourier, Hough, etc. — or outstep (Fedotov et al., 2000) (Fedotov et al., 2005).

Property of Trace-transform. Trace-transform proves convenient to study objects' movements and similarity transform within the retina.

Let us briefly consider how image $\mathbf{T}(F)$



a) b)

Figure 3: The experiment on a doubly-connected and single-connected images and corresponding Trace-matrices

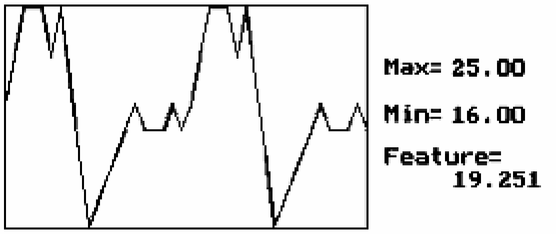


Figure 4: The result of applying Diametrical functional to a Trace–matrix and the calculation of triple feature by means of Circus functional

changes after initial image F has been shifted and rotated. If the original image rotates, its Trace-image shifts along the horizontal axis 0θ. If the original image shifts to a certain vector, its Trace-image undergoes changes as follows. For convenience they are to be stated in terms of Trace-matrices. Columns remain unchanged and stay in their places, though may shift up or down. Shift vector specifies numbers a and b such that a column with coordinate θ_i shifts vertically to $a\cos(\theta_i - b)$. It should be noted that the description is to be totally rigorous if only we consider Trace-matrix continuous, i.e., i and j are continuous parameters.

3 IMAGE PREPROCESSING BASED ON TRACE-TRANSFORM

3.1 Nonlinear filtration

Through a Trace-transform, one can implement nonlinear filtration of image to reduce its noise and quantization.

Figure 2,a shows the original image, while Figure 2,b shows the same image distorted by additive noise. If we try to obtain their Trace-transforms using the functionals considered above we get different Trace-matrices, the latter containing information both about original image and the noise.

For getting information only about the original image we propose to use more complicated **T**-functionals. For example, correspondent **T**-functional (1) let us to get information about original image excepting noise information:

$$a'_{ij} = \begin{cases} 0\,, \text{if } a_{ij} = 0 \\ 10\lg a_{ij}\,, \text{ if } a_{ij} > 0 \end{cases} \quad (1)$$

where a_{ij} are number of intersections scanning line of given image.

The images and its Trace-transforms obtained by functional (1) depict in Figure 2. One can see that pure (Figure 2,a) and noisy (Figure 2, b) images have identical Trace-matrices.

This approach was successfully applied in diagnostics of welded joints and pores. Through the Trace-transform, we get a matrix of an original dimension, which elements almost unexceptionally differ from zero if they correspond to a pore.

Other types of T-functional have been found to replace certain types of image pre-processing, such as inversion, and filling closures.

3.2 Evaluation of objects' relative position and number

Let us consider a valuable feature of Trace-transforms to tackle certain problems of image analysis normally solved structurally: through segmentation, or evaluation of objects' relative position and number. Figure 3,b shows an object to be viewed as having a single-connected, and its Trace-matrix, Figures 3,a showing segmented images and their Trace-matrices.

The example, which proves structuralistic features of Trace-transform, applies to the field of technical diagnostics. A problem of recognizing welded joints' pores was being solved based on the results of their radiographic inspection. Figure 3 shows one of the defections typical for welded joints — spherical pores. Instructions on radiographic inspection request that chains of pores be differentiated from single pores and other defections, such as slag inclusions, incomplete penetration, etc.

Moreover, this type of defection is suggested to be differentiated according to the relative position of pores within the chain, as well as to the number, and the size (the diameter) of pores.

It should be noted that one could enhance non-linear filtration applying the filtering capabilities of both the Trace-functional and other functional of triple structure described below.

4 TRIPLE FEATURES

The most valuable feature of the Trace-transform is that it proves a source of a new class of recognition features which have a triple structure (triple features) (Fedotov, 1990) (Fedotov et al., 1995).

Analysis of great many formulas of stochastic geometry, as well as well-known geometric transformations — those of Fourier, Chough, Radon — demonstrate that they can be represented as a composition of three functionals. From this conclusion, the authors construct recognition features as a composition of three functionals. Their recognition power matters when solving classification problems.

Let us consider formation of triple features which are a consecutive composition of three functionals:

$$\Pi(F) = \mathbf{\Theta} \circ \mathbf{P} \circ \mathbf{T}(F \circ L(\theta, p, t)) . \qquad (2)$$

Each functional $\mathbf{\Theta}$, P and T effects the function of one variable θ, p and t correspondingly. For each of the three functionals it is easy to think out dozens of various concretizations which comply with the conditions required. Hence, we may at once get thousands of new features invariant to motions.

Functional T, corresponding to a Trace-transformation, has been above considered in detail. In a discrete variant of computation the result of the transformation, or the Trace-transform, $\mathrm{T}(F \circ l(\theta, p, t))$ is a matrix, which elements are, say, values of brightness parameter for image F at the intersections with the scanning line $l(\theta, p)$. Parameters of the scanning line p specify the position of the element within the matrix. Computation of feature to follow involves a consecutive processing of the matrix columns with the help of functional P, which we call diametrical. Functional "Norm", a standard Euclidean norm $\mathrm{P}h = \sqrt{\int h^2(p)dp}$ has been used as functional P, other instances of diametrical functionals applied may be the functional called "Max", which is the maximum value of the function in a Trace–matrix column; and the "Mid" functional. It is a standard middle point computed through

$$\mathrm{P}h = \frac{\int ph(p)dp}{\int h(p)dp}. \qquad (3)$$

The result of applying P ("Norm") functional to a Trace–matrix (Figure 1,b) is a 2π - periodic curve shown in Figure 4.

Next stage is to perform transformations on the curve with the help of $\mathbf{\Theta}$ functional, which we call a circus within N. The "Log" functional has been used as a variant of $\mathbf{\Theta}$ functional, being computed through

$$\Theta h = \int \left| \ln h(\theta) + 1 \right| d\theta. \qquad (4)$$

The triplex recognition features considered may be computed through a highly parallel process. Like features formed by neuron nets, the given features have no pre-assigned meanings, their selection being realized during a machine experiment, considering their being useful for classification only.

5 FUNCTIONALS INVARIANT AND SENSITIVE TO THE AFFINE TRANSFORMATIONS

One can obtain dozens of features using three-functional structure. But it is a fact that features useful for practical application can be classified into two types: invariant to affine transformation of an image and sensitive to them.

Let us provide more strict definitions of the invariant and sensitive functionals.

We call functional Ξ invariant if $\Xi(u \circ (x+b)) = \Xi u$ for all $b \in R$.

We call functional Z sensitive if $\mathrm{Z}(u \circ (x+b)) = \mathrm{Z}u - b$ for all $b \in R$.

The following theorem is proven: if functional $\mathrm{T}, \mathrm{P}, \Theta$ are invariant, then feature $\Pi(F)$ does not depend upon the group of all motions.

Therefore, we can rapidly and successfully find identical images in the sequence of images regardless of their affine transformations with the help of invariant functionals (for example, when we investigate the structure of micro objects, we determine the structure elements regardless of their location or scaling up the microscope).

We use sensitive functionals to compute the coordinates of object motions (it can be helpful in machine vision system for exact positioning of robot tools). Efficiency of designed algorithms and methods was proved by means of experiments in the field of nanotechnology and in technical and medicine diagnostic systems designing.

Thus, triple feature theory allows us to create new features of two types: those are invariant to affine transformation for given images (such as translation, rotation and scale transforms) and their linear deformations, and those are sensitive to affine transformations.

One can easily propose tens of different specific realizations of each of the three functionals which satisfy the required conditions. Hence, without much effort, we get thousands of new features which are invariant or sensitive to image motion. This only proves the value of the considered theory for the problems of pattern recognition with multiple structure of the classes, like the problem of the recognition of hieroglyphs or textures. In the following, some functionals are presented which we use to solve different problems and which are implemented on a PC.

a) Invariant functionals:

1. Maximum value of the function.
2. The number of extrema of the function.
3. Total variation of the function.
4. Standard Euclidean norm of the function.
5. Any functional determined by the distribution of the values of the function.

b) Sensitive functionals:

1. Standard center of gravity of the masses calculated by formula (3).
2. The phase of the second Fourier harmonics of the function.
3. Absolute value of the Fourier coefficient of the third harmonics.
4. Probability-theoretic median.

6 CONCLUSION

Trace-transform has been considered, the latter applying all-direction scanning.

The most valuable property of the Trace-transform is that it establishes the new class of recognition features with a triple structure (triple features).

It is possible to form great many features (in fact, thousands of them), as it needs to recognize patterns with a great number of classes. A vast number of features helps to understand image better and to increase recognition flexibility and fidelity.

Besides, it has been established that the theory based on triple features yields a stable result when analyzing biological micro and nano objects, and, in particular, when recognizing images in the field of nanotechnology. Results have been obtained in successful recognition of leucocytes and erythrocytes, which can be found in (Fedotov et al., 2000). Moreover, the theory of triple features stated above has been tested on and proved suitability for technical diagnostics and flaw detection problems. Applicability of the given ideas has been experimentally proved in (Fedotov et al., 2002).

The theory created allows us to obtain features independent of object motions or linear deformations. Additionally, we can use the theory to obtain features which depend on the transformations mentioned above in a simple way giving us a possibility to compute the parameters of motions and motions or linear deformations transformed objects (it can be helpful in machine vision system for exact positioning of robot tools).

A composition of three functionals is applicable at the same time to construct recognition features and to perform non-linear filtration, to reduce noise within the image and to segment the objects, to smooth the image etc. It should be noted that one could enhance non-linear filtration applying the filtering capabilities of both the Trace-functional and another functional of triple structure. Having the opportunity to construct features and to perform image pre-processing simultaneously and by the same technique, one can make recognition system faster, pre-processing and feature construction being realized at the same stage of the scanning.

REFERENCES

Fedotov N.G., 1990. *Metody Stokhasticheskoi Geometrii v Raspoznavanii Obrazov (Methods of Stochastic Geometry in Patters Recognition)*, Moscow, Radio i Svjaz.

Fedotov N.G., Kadyrov A.A., 1995. *Image scanning in machine vision leads to new understanding of image.* In Proc. of 5th International Workshop on Digital In Processing and Computer Graphics, Proc. International Society for Optical Engineering (SPIE), Vol. 2363, pp. 256-261.

Helgason S., 1980. *The Radon Transform*, Birkhauser, Basel and Boston, Massachusetts.

Fedotov N.G., Shulga L.A., 2000. *New Theory of Pattern Recognition on the Basis of Stochastic Geometry*. In WSCG'2000 Conference Proceedings, Vol. 1, p. 373-380.

Fedotov N.G., Shulga L.A., 2004. *Feature Generation and Stochastic Geometry*. In Proc. of the 4th International Workshop on Pattern Recognition in Information Systems, PRIS'2004, Porto, Portugal, p. 169-175.

Fedotov N.G., Shul'ga L.A., Roy A.V., 2005. *Visual Mining for Biometrical System Based on Stochastic Geometry Pattern Recognition and Image Analysis*, Vol. 15, No. 2, 2005, pp. 389–392.

Fedotov N.G., Nikiforova T.V., 2002. *Technical defectoscopy based on new pattern recognition theory*, Izmeritel'naya tekhnika (Measurement technics), 2002, No. 12, pp. 27-31.