A DESIGN METHOD OF TWO-DIMENSIONAL LINEAR PHASE FIR FILTERS USING FRITZ JOHN'S THEOREM

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Abstract: This paper presents a design method of 2-dimensional (2-D) FIR filters by successive projection (SP) method using multiple extreme frequency points based on Fritz John's theorem. The proposed method enables an update of coefficients using multiple extreme frequency points by Fritz John's theorem. Moreover, we also present two methods as how to choose the extreme frequency point for the update coefficients. As a result, the solution converges less iteration number and computing time than the previous method.

1 INTRODUCTION

The SP method proposed by A. A. -Taleb et al. in 1984 (Taleb and Fahmy, 1984) has been applied to the design problem of many filters, such as FIR filters with complex desired frequency response, IIR filters, a class of time-constrained FIR filters, FIR filters which approximate the amplitude characteristics and the step response simultaneously (Sugita and Aikawa, 2004). In the SP method which is an iterative approximation method, only one extreme frequency point at which the deviation from the given specification is maximized is used in the update of the filter coefficients. Hence, this algorithm is extremely simple since it only requires the search for the maximum of the error function over a closed frequency region in each iteration. However, because this method is used only one extreme frequency point in the update of the filter coefficients, this algorithm requires a large iteration number to satisfy the given specification. In (Sugita and Aikawa, 2004), authors proposed a new algorithm to reduce the iteration number for designing 1-dimensional (1-D) filter which satisfies the given specification. This method uses multiple extreme frequency points in order to update the filter coefficients by Fritz John's theorem. As a result, it is possible to reduce computing time further than the conventional SP method.

In this paper, we propose a design method of 2-D FIR filters by SP method using multiple extreme frequency points based on Fritz John's theorem. The proposed method is possible to reduce the iteration number and computing time further than the conventional SP method by using multiple extreme frequency points for the updating coefficients. Moreover, we propose two selection methods of the multiple extreme frequency points for updating coefficients.

2 DESIGN FORMULATION AND SOLUTON BY SP METHOD

The amplitude characteristics of 2-D linear phase FIR filters can easily be shown (Taleb and Fahmy, 1984) to have the form

$$H\left(\omega_1, \omega_2\right) = \sum_{i=1}^{N} a_i \phi_i(\omega_1, \omega_2). \quad (1)$$

Where the coefficients $a_i\,(i = 1, 2, \cdots, N)$ are related to the impulse response samples of the filter, ϕ_i are frequency dependent functions having a form depending on the type of symmetries (e.g., half plane, quadrantal, or octagonal) imposed on the amplitude characteristics and N is an integer which is defined by the filter mask size.

Then, the design problem considered here is to find the coefficients a_i satisfying

$$\left| D\left(\omega_1, \omega_2\right) - \sum_{i=1}^{N} a_i \phi_i(\omega_1, \omega_2) \right| \le \lambda\left(\omega_1, \omega_2\right). \quad (2)$$

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Whrere $D(\omega_1, \omega_2)$ is the desired amplitude characteristics and $\lambda(\omega_1, \omega_2)$ is the positive maximum allowable deviation from the desired amplitude characteristics.

As a result, the iterative algorithm of A. A. -Taleb et al. is

$$a_i^{n+1} = a_i^n + \frac{\left[\left|e_p^n\right| - \lambda(\omega_1, \omega_2)_p\right] \operatorname{sign}(e_p^n)}{\sum_{m=1}^{N} \phi_m^2(\omega_1, \omega_2)_p} \phi_i(\omega_1, \omega_2)_p, \tag{3}$$

where

$$e_p^{n+1} = D(\omega_1, \omega_2)_p - \sum_{i=1}^{N} a_i^{n+1} \phi_i(\omega_1, \omega_2)_p. \tag{4}$$

The detail of this algorithm has been presented in (Taleb and Fahmy, 1984).

It is clear from (3) that this algorithm only requires a search for the maximum error function over a closed frequency region. However, enormous amount of iteration numbers are necessary for research the solution which satisfies the given specification, because only one extreme frequency point at which the deviation from the given specification is maximized is used in the update of the filter coefficients.

3 NEW ALGORITHM

In this section, the update method of coefficients using multiple extreme frequency points based on Fritz John's theorem is described.

[Fritz John's Theorem]

Let X be a nonempty open set in n-dimensional Euclidean space E_n, and let $f : E_n \to E_1$, $g_i : E_n \to E_1$ for $i = 1, ..., m$. Consider Problem P to

Minimize $f(\boldsymbol{x})$

subject to $g_i(\boldsymbol{x}) \leq 0$ for $i = 1, ..., m$

$\boldsymbol{x} \in X$

Let $\bar{\boldsymbol{x}}$ be a feasible solution, and let $I = \{i : g_i(\bar{\boldsymbol{x}}) = 0\}$. Furthermore, suppose that g_i for $i \notin I$ is continuous at $\bar{\boldsymbol{x}}$, that f_i and g_i for $i \in I$ are differentiable at $\bar{\boldsymbol{x}}$. If $\bar{\boldsymbol{x}}$ locally solves Problem P, then there exist scalars u_0, u_i for $i \in I$ such that

$$u_0 \nabla f(\bar{\boldsymbol{x}}) + \sum_{i \in I} u_i \nabla g_i(\bar{\boldsymbol{x}}) = \mathbf{0} \tag{5a}$$

$$u_0, u_i \geq 0 \quad \text{for } i \in I \tag{5b}$$

$$(u_0, \boldsymbol{u}_I) \neq (0, \mathbf{0}) \tag{5c}$$

where $\boldsymbol{u}_I$ is the vector whose components are u_i for $i \in I$. Furthermore, if g_i for $i \notin I$ is also differentiable at $\bar{\boldsymbol{x}}$, then the Fritz John conditions can be written in the following equivalent form where $\boldsymbol{u} = (u_1, ..., u_m)^t$.

$$u_0 \nabla f(\bar{\boldsymbol{x}}) + \sum_{i=1}^{m} u_i \nabla g_i(\bar{\boldsymbol{x}}) = \mathbf{0} \tag{5d}$$

$$\boldsymbol{u}_i g_i(\bar{\boldsymbol{x}}) = 0 \text{ for } i = 1, ..., m \tag{5e}$$

$$u_0, u_i \geq 0 \text{ for } i = 1, ..., m \tag{5f}$$

$$(u_0, \boldsymbol{u}) \neq (0, \mathbf{0}) \tag{5g}$$

■

A proof of this theorem can be found in (Bazaraa and Shetty, 1979).

We consider the optimization problem as following in order to use Fritz John's theorem for design problem of section 2. :

$$\text{Minimize} \quad f\left(\boldsymbol{a}^{n+1}\right) = \sum_{i=1}^{N} \left(a_i^{n+1} - a_i^n\right)^2 \tag{6}$$

Subject to

$$\begin{aligned} g_1\left(\boldsymbol{a}^{n+1}\right) &= \left|e_1^{n+1}\right| - \lambda(\omega_1, \omega_2)_1 \leq 0 \\ g_2\left(\boldsymbol{a}^{n+1}\right) &= \left|e_2^{n+1}\right| - \lambda(\omega_1, \omega_2)_2 \leq 0 \\ &\vdots \\ g_l\left(\boldsymbol{a}^{n+1}\right) &= \left|e_l^{n+1}\right| - \lambda(\omega_1, \omega_2)_l \leq 0 \\ &\vdots \\ g_m\left(\boldsymbol{a}^{n+1}\right) &= \left|e_m^{n+1}\right| - \lambda(\omega_1, \omega_2)_m \leq 0. \end{aligned} \tag{7}$$

Where the error function e_l^{n+1} in lth condition $g_l\left(\boldsymbol{a}^{n+1}\right)$ is

$$e_l^{n+1} = D(\omega_1, \omega_2)_l - \sum_{i=1}^{N} a_i^{n+1} \phi_i(\omega_1, \omega_2)_l \tag{8}$$

from (4). For simplicity, we put λ_l with $\lambda(\omega_1, \omega_2)_l$.

If $\boldsymbol{a}^{n+1}$ is an locally optimal solution of this problem, with Fritz John's theorem, we get

$$u_0 \nabla f\left(\boldsymbol{a}^{n+1}\right) + \sum_{l=1}^{m} u_l \nabla g_l\left(\boldsymbol{a}^{n+1}\right) = \mathbf{0} \tag{9a}$$

$$(u_0, \boldsymbol{u}) \neq (0, \mathbf{0}). \tag{9b}$$

Substituting (6) and (7) in (9), we get

$$a_i^{n+1} = a_i^n + \frac{1}{2u_0} \sum_{l=1}^{m} u_l \phi_i(\omega_1, \omega_2)_l. \tag{10}$$

Moreover, substituting (10) in (7), and considering equal to zero give Lagrange multipliers :

$$\begin{pmatrix} u_1/u_0 \\ u_2/u_0 \\ \vdots \\ u_l/u_0 \\ \vdots \\ u_m/u_0 \end{pmatrix} = \boldsymbol{G}^{-1} * \begin{pmatrix} 2\left(e_1^n - \lambda_1\right) \\ 2\left(e_2^n - \lambda_2\right) \\ \vdots \\ 2\left(e_l^n - \lambda_l\right) \\ \vdots \\ 2\left(e_m^n - \lambda_m\right) \end{pmatrix}. \tag{11}$$

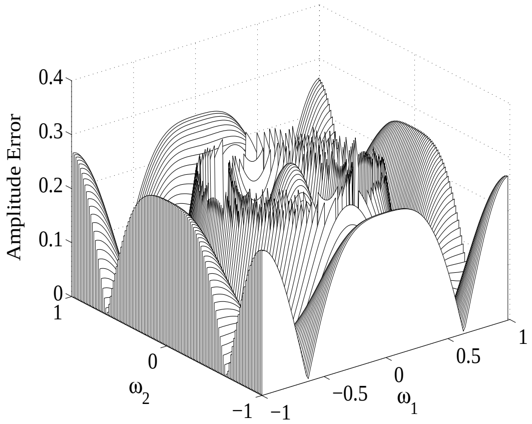

Figure 1: The amplitude error characteristics of 2-D FIR filters

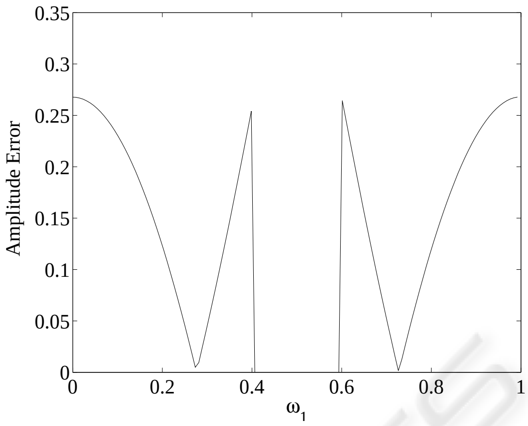

Figure 2: The amplitude error characteristics on $\omega_2 = 0$ axis of 2-dimensional FIR filter in Fig. 1

Where

$$\boldsymbol{G} = \begin{pmatrix} G_{1,1} & G_{1,2} & \cdots & G_{1,k} & \cdots & G_{1,m} \\ G_{2,1} & G_{2,2} & \cdots & G_{2,k} & \cdots & G_{2,m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ G_{l,1} & G_{l,2} & \cdots & G_{l,k} & \cdots & G_{l,m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{m,1} & G_{m,2} & \cdots & G_{m,k} & \cdots & G_{m,m} \end{pmatrix} \tag{12}$$

and (l, k)-th component $G_{l,k}$ is given by

$$G_{l,k} = \sum_{i=1}^{N} \phi_i \left(\omega_1, \omega_2\right)_l \cdot \phi_i \left(\omega_1, \omega_2\right)_k . \tag{13}$$

Let's $\Delta_{l,k}$ be (l, k)-th cofactor of matrix $\boldsymbol{G}$. Then, the iterative algorithm using Fritz John's theorem is described as

$$a_i^{n+1} = a_i^n + \frac{1}{|\boldsymbol{G}|} \sum_{l=1}^{m} \sum_{k=1}^{m} (|e_k^n| - \lambda_k) \text{sign}(e_k^n) \Delta_{l,k} \phi_i \left(\omega_1, \omega_2\right)_l \tag{14}$$

by substituting (11) in (10). Where m is the number of extreme frequency points used for the updating coefficients and must not exceed the number N of unknown coefficients.

By the way, because it is well known that the Haar condition (Cheney, 1966) consists in designing 1-D filter, the extreme frequency points exist at least $N+1$ point, where N is the number of unknown coefficient. However, the amplitude error characteristic of 2-D filter is very complicated as shown in Fig. 1. Especially, it is clear from Fig. 1 that the detection of the extreme frequency points in band-edge is very difficult. In addition, it is known that N-dimensional linear space of approximation function (1) does not generally satisfy the Haar condition (Kamp and Thiran, 1975). Therefore, because the extreme frequency points more than the number of unknown coefficients are found, there is a possibility that the proposed algorithm does not converge. Hence, we propose two methods of finding the multiple extreme frequency points less than the number of unknown coefficients as follows.

(I) : The maximum extreme frequency point in each approximation band is used.

(II) : The maximum extreme frequency point in all approximation bands and the extreme frequency points in an arbitrary cross section are used.

In method (I), since the number of approximation band of a 2-D linear phase Lowpass or Highpass FIR filter is two, the number of the extreme frequency points is always less than the number N of unknown coefficient.

In method (II), the amplitude error characteristic of 2-D filter in an arbitrary cross section behave like one of 1-D filter, as shown in Fig. 2. Accordingly, it is clear that the number of extreme frequency points on this axis is always less than the number of unknown coefficients.

4 RESULT

In this section, we show some examples of 2-D filter designed by the proposed method and the conventional SP method (Taleb and Fahmy, 1984). All algorithm used for the design examples are corded in Visual $C^{++}6.0$ and run on a Pentium 4/3.06GHz PC. In proposed method, the simulation was carried out for both of above method (I) and (II). In all of the examples, an initial coefficients vector $\boldsymbol{a}^0$ is taken equal to zero.

First, we will design a circularly symmetric lowpass filter of specification as following.

Table 1: Comparison of the number of the iteration

			Conventional SP		Proposed Method (I)		Proposed Method (II)	
filter mask	M	deviation λ	Iteration number	Time m : sec.	Iteration number	Time m : sec.	Iteration number	Time m : sec.
5×5	100	0.268	4401	1 : 02	2657	0 : 37	1989	0 : 28
7×7	100	0.126	2438	0 : 56	1866	0 : 43	938	0 : 21
9×9	100	0.118	792	0 : 27	481	0 : 16	203	0 : 10
25×25	200	0.030	588	7 : 49	347	5 : 47	198	3 : 34

filter mask 5×5

$$D(\omega)=\begin{cases} 1 \cdots 0 \leq \sqrt{\omega_1^2+\omega_2^2} \leq 0.4\pi \\ 0 \cdots 0.6\pi \leq \sqrt{\omega_1^2+\omega_2^2} \leq \pi \end{cases}$$

The amplitude characteristics of the filter obtained is presented in Fig.3. The maximum amplitude error of the resulting filter is 0.268 in passband and 0.268 in stopband. These results are equal to A. A. Taleb et al.'s results (Taleb and Fahmy, 1984). In evaluating the design errors, the frequency ω_1 and ω_2 are sampled at the size $\pi/100$. In our proposed algorithm, the design took 1989 iterations and 28 seconds to satisfy the given specification. If this filter is designed by conventional SP method, the iteration number of 4401 iterations and computing time of 62 seconds are required. Clearly, the proposed method has provided a very significant savings.

Table 1 shows a comparison of the algorithm due to conventional SP method (Taleb and Fahmy, 1984) and the proposed method in the design of the other examples. It is proven from table 1 that the proposed method is much faster than the conventional SP method in all of the examples. Moreover, the iteration number and computing time necessary for research the solution decrease by increasing of extreme frequency points used for the updating coefficients. Therefore, the number of the iteration can be drastically reduced by increasing the number of extreme frequency points also in the case of a 2-D filter.

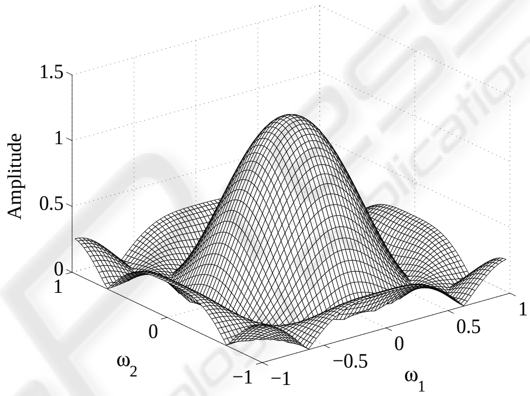

Figure 3: The amplitude characteristics of filter mask (5×5)

5 CONCLUSION

In this paper, we proposed a design method of 2-D FIR filter by SP method using multiple extreme frequency points based on Fritz John's theorem. In case of the design of 2-D FIR filter, the determination method of the extreme frequency point for the updating coefficients becomes a problem. We proposed two determination methods for multiple extreme frequency points used for the updating coefficients. As a result, the proposed method is possible to reduce the number of the iteration and the computing time necessary for research the solution which satisfies the given specification, further than the conventional SP method. We confirmed that the proposed method converged in many examples.

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