# TRACKING-CONTROL INVESTIGATION OF TWO X4-FLYERS

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Abstract: Two models of mini-flying robots with four rotors called X4-flyer presented and studied for the stabilization. Both cases with and without motion planning are proposed in this paper. The first is called inertial model with axes orientation and the second is called the inertial model without axes orientation. The control algorithm of the X4-flyer is based on the Lyapunov method and obtained using the backstepping techniques. This enabled to stabilize the engine in hovering and to generate its trajectory. The system behavior using the proposed control law is described through numerical simulations.

## 1 INTRODUCTION

The automatic control of flying machines has attracted the attention of many researches in the past few years. Generally, the control strategies are based on simplified models which have both a minimum number of states and a minimum number of input. These reduced models should retain the main features that must be considered when designing control laws for real aerial vehicles. The rotorcraft is one the most complex flying machines. Its complexity is due to the versatility and manoeuvrability to perform many types of tasks (Castillo et al., 2004). Very little attention has been done on the development of aerial robotic platforms (Altug, 2003) (Altug et al., 2003) (Altug et al., 2002) (Hamel et al., 2002) (Zhang, 2000). Such platforms have considerable commercial potential for surveillance and inspection roles in dangerous environments.

Modelling and controlling aerial vehicles (blimps, mini rotorcraft) are the principal preoccupation of our laboratory (LSC). In this topic, a mini-UAV is developped by the LSC-group taking into account industrial constraints. The aerial flying engine could not exceed $2kg$ in mass, a wingspan of $50cm$ with a $30mn$ flying-time (see figure 1). Within this optic, it can be held that our system belongs to a family of mini-UAV. It is an autonomous hovering system, capable of vertical takeoff, landing, lateral motion and quasi-stationary (hover or near hover) flight conditions. Compared to helicopters (Altug, 2003) (Altug et al., 2003) (Altug et al., 2002), the four rotors rotorcraft called X4-flyer has some advantages (Hamel et al., 2002) (Pound et al., 2002): given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight, to cancel. An X4-flyer operates as an omnidirectional UAV. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in $x$ direction or in $y$ direction, is achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations.

Several recent work was completed for the design and control in pilot-less aerial vehicles domain such that Quadrotor (Altug, 2003) (Altug et al., 2003) (Altug et al., 2002), X4-flyer (Hamel et al., 2002), mesicopter (Kroo and Printz, ) and hoverbot (Borenstein, ). Also, related models for controlling the VTOL aircraft are studied by Hauser and al (Hauser et al., 1992). A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotors vertical take-off and landing (VTOL) was studied by Hamel (Hamel et al., 2002) where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotors rotorcraft is also studied and tested by Castillo (Castillo


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et al., 2004) where the nested saturation algorithm is used and application of the theory of flat systems by Beji et al (Beji et al., 2004).

Figure 1: Conceptual form of the four rotors rotorcraft.

In this paper, the backstepping controllers and motion planning are combined to stabilize the helicopter by using the point to point steering stabilization. After having presented the study of modeling and the description of the configuration in the second section. Third section describes the dynamics of the system which treats the two models with and without axes orientation. Backstepping controllers is described for two models of the X4-flyer in the fourth section. A strategy to solve the tracking problem through point to point steering is shown in the fifth section. In the sixth section simulation results are introduced for two models. Finally, conclusion and future work are given in the last section.

## 2 CONFIGURATION DESCRIPTION AND MODELLING

Unlike regular helicopters that have variable pitch angles, an engine has fixed pitch angle rotors and the rotor speeds are controlled to produce the desired lift forces. Basic motions of the four rotors rotorcraft can described using the figure 2. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in $x$ direction or in $y$ direction, is not achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations (case of the X4-flyer). But, two engines of direction are used to permute between the $x$ and $y$ motion.

We consider a local reference airframe $\Re_G = \{G, E_1^g, E_2^g, E_3^g\}$ attached to the mass center $G$ of the vehicle. The mass center is located at the intersection of the two rigid bars, each of which supports two motors. Equipments (controller cartes, sensors, etc.)

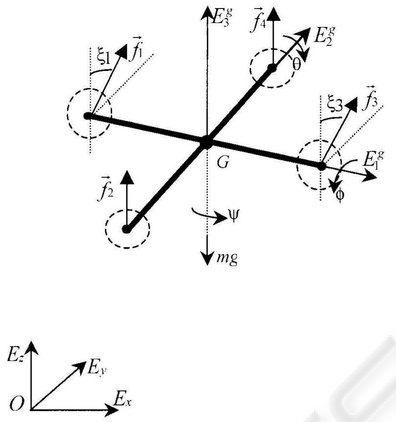



Figure 2: 3D X4-flyer model.

onboard are placed not far from $G$. The inertial frame is denoted by $\Re_O = \{O, E_x, E_y, E_z\}$. A body fixed frame is assumed to be at the center of gravity of the X4-flyer, where the $z$ axis is pointing upwards. This body axis is related to the inertial frame by a position vector $(x, y, z)$ and 3 Euler angles $(\theta, \phi, \psi)$ representing pitch, roll and yaw respectively. A Euler angle representation given in (1) has been chosen.

$$R = \begin{pmatrix} C_\psi C_\theta & C_\theta S_\psi & -S_\theta \\ S_\phi C_\psi S_\theta - S_\psi C_\phi & S_\theta S_\psi S_\phi + C_\psi C_\phi & C_\theta S_\phi \\ S_\theta C_\psi C_\phi + S_\psi S_\phi & C_\phi S_\theta S_\psi - C_\psi S_\phi & C_\theta C_\phi \end{pmatrix} \tag{1}$$

Where $C_\theta$ and $S_\theta$ represent $cos\,\theta$ and $sin\,\theta$ repectively.

Each rotor produces moments as well as vertical forces. These moments have been experimentally observed to be linearly dependent on the forces for low speeds. There are four/five input forces and six output states $(x, y, z, \theta, \phi, \psi)$ therefore the X4-flyer is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counterclockwise, in order to balance the moments and produce yaw motions as needed.

In the present work, two X4-flyer models are presented, the first is called the inertial model with axes orientation, the second one is the inertial model without axes orientation. For the model without axes orientation, the rotors 2 and 4 are actuated in clockwise direction, the remain rotors, the rotors 1 and 3 are in the contrary actuated in the inverse direction in order to guarantee total balance in yaw (figure 4). The main feature of the presented X4-flyer (called the XSF) in comparison with the existing quadrirotors, is the swiveling of the actuators supports 1 and 3 around the axis of pitching (angles $\xi_1$ and $\xi_3$). This swiveling ensures either the horizontal rectilinear motion or the rotational movement around the yaw axis or a combination of these two movements which gives the turn (see the figure 3), as well as the direction of rotation of the rotors implies that rotors 1 and 2 turn clockwise and rotors 3 and 4 turn in the contrary direction of the needles of a watch.

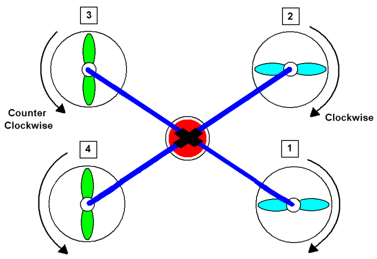


Figure 3: Rotor rotations with canceled yaw motions.

# 3 MOTION DYNAMIC

We consider the translation motion of $\Re_G$ with respect to $(wrt)$ $\Re_O$. The position of the center of mass $wrt$ $\Re_O$ is defined by $\overline{OG} = (x\ y\ z)^T$, its time derivative gives the velocity $wrt$ to $\Re_O$ such that $\frac{d\overline{OG}}{dt} = (\dot{x}\ \dot{y}\ \dot{z})^T$, while the second time derivative permits to get the acceleration $\frac{d^2\overline{OG}}{dt^2} = (\ddot{x}\ \ddot{y}\ \ddot{z})^T$. In the following, the model with axes orientation is described, then the model without axes orientation is given.

## 3.1 Dynamic Motion of the Model with Axes Orientation

Currently, the model is a simplified one's. The constraints and the gyroscopic torques are neglected. The aim is to control the engine vertically $(z)$ axis and horizontally according to $x$ and $y$ axis. The dynamics of the vehicle, represented on figure 2, is modelled by the system of equations (2), (Beji et al., 2005).

$$\begin{array}{l} m\ddot{x} = S_\psi C_\theta u_2 - S_\theta u_3 \\ m\ddot{y} = (S_\theta S_\psi S_\phi + C_\psi C_\phi)\, u_2 + C_\theta S_\phi u_3 \\ m\ddot{z} = (S_\theta S_\psi C_\phi - C_\psi C_\phi)\, u_2 + C_\theta C_\phi u_3 - mg \end{array} \tag{2}$$

Where $m$ is the total mass of the vehicle. The vector $u_2$ and $u_3$ combines the principal non conservative forces applied to the engine airframe including forces generated by the motors and drag terms. Drag forces and gyroscopic due to motors effects will be not considered in this work. The lift (collective) force $u_3$ and the direction input $u_2$ are such that

$$\begin{pmatrix} 0 \\ u_2 \\ u_3 \end{pmatrix} = f_1 \acute{e}_1 + f_2 e_2 + f_3 \acute{e}_3 + f_4 e_4 \tag{3}$$

with $f_i = k_i \omega_i^2, k_i > 0$ is a given constant and $\omega_i$ is the angular speed resulting of motor $i$. Let

$$\begin{array}{c} \acute{e}_1 = \begin{pmatrix} 0 \\ S_{\xi_1} \\ C_{\xi_1} \end{pmatrix}_{\Re_G} ; \acute{e}_3 = \begin{pmatrix} 0 \\ S_{\xi_3} \\ C_{\xi_3} \end{pmatrix}_{\Re_G} \\ e_2 = e_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{\Re_G} \end{array} \tag{4}$$

Then we deduce:

$$\begin{array}{l} u_2 = f_1 S_{\xi_1} + f_3 S_{\xi_3} \\ u_3 = f_1 C_{\xi_1} + f_3 C_{\xi_3} + f_2 + f_4 \end{array} \tag{5}$$

$\xi_1$ and $\xi_3$ are the two internal degree of freedom of rotors 1 and 3, respectively. These variables are controlled by dc-motors and bounded $-20^o \leq \xi_1, \xi_3 \leq +20^o$. $e_2$ and $e_4$ are the unit vectors along $E_3^g$ which imply that rotors 2 and 3 are identical of that of a classical Quadrotor (not directional).

## 3.2 Rotational Motion of the Model with Axes Orientation

The rotational motion of the X4 bidirectional flyer will be defined $wrt$ to the local frame but expressed in the inertial frame. According to Classical Mechanics, and knowing the inertia matrix $I_G = diag\,(I_{xx}, I_{yy}, I_{zz})$ at the centre of the mass.

$$\begin{array}{l} \ddot{\theta} = \frac{1}{I_{xx} C_\phi} (\tau_\theta + I_{xx} S_\phi \dot{\phi} \dot{\theta}) \\ \ddot{\phi} = \frac{1}{I_{yy} C_\theta C_\phi} (\tau_\phi + I_{yy} S_\phi C_\theta \dot{\phi}^2 + I_{yy} S_\theta C_\phi \dot{\theta} \dot{\phi}) \\ \ddot{\psi} = \frac{\tau_\psi}{I_{zz}} \end{array} \tag{6}$$

With the three inputs in torque

$$\begin{array}{l} \tau_\theta = l\,(f_2 - f_4) \\ \tau_\phi = l\,(f_1 C_{\xi_1} - f_3 C_{\xi_3}) \\ \tau_\psi = l\,(f_1 S_{\xi_1} - f_3 S_{\xi_3}) \end{array} \tag{7}$$

where $l$ is the distance from $G$ to the rotor $i$. The equality from (6) is ensured, meaning that

$$\ddot{\eta} = \Pi_G\,(\eta)^{-1} \left[\tau - \dot{\Pi}_G\,(\eta)\,\dot{\eta}\right] \tag{8}$$

With $\tau = (\tau_\theta, \tau_\phi, \tau_\psi)^T$ as an auxiliary inputs.
And

$$\Pi_G\,(\eta) = \begin{pmatrix} I_{xx} C_\phi & 0 & 0 \\ 0 & I_{yy} C_\phi C_\theta & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \tag{9}$$

As a first step, the model given above can be input/output linearized by the following decoupling feedback laws

$$\begin{array}{l} \tau_\theta = -I_{xx} S_\phi \dot{\phi} \dot{\theta} + I_{xx} C_\phi \tilde{\tau}_\theta \\ \tau_\phi = -I_{yy} S_\phi C_\theta \dot{\phi}^2 - I_{yy} S_\theta C_\phi \dot{\theta} \dot{\phi} + I_{yy} C_\theta C_\phi \tilde{\tau}_\phi \\ \tau_\psi = I_{zz} \tilde{\tau}_\psi \end{array} \tag{10}$$

and the decoupled dynamic model of rotation can be written as

$$\ddot{\eta} = \tilde{\tau} \tag{11}$$

with $\tilde{\tau} = (\tilde{\tau}_\theta \tilde{\tau}_\phi \tilde{\tau}_\psi)^T$

Using the system of equations (2) and (11), the dynamic of the system is defined by

$$\begin{array}{l} m\ddot{x} = S_\psi C_\theta u_2 - S_\theta u_3 \\ m\ddot{y} = (S_\theta S_\psi S_\phi + C_\psi C_\phi)\, u_2 + C_\theta S_\phi u_3 \\ m\ddot{z} = (S_\theta S_\psi C_\phi - C_\psi C_\phi)\, u_2 + C_\theta C_\phi u_3 - mg \\ \ddot{\theta} = \tilde{\tau}_\theta; \quad \ddot{\phi} = \tilde{\tau}_\phi; \quad \ddot{\psi} = \tilde{\tau}_\psi \end{array} \tag{12}$$

## 3.3 Dynamic Motion of the Model without Axes Orientation

We follows the same steps as the model with axes orientation and finally we finds for the dynamics of the X4-flyer without the axes orientation:

$$\begin{array}{l} m\ddot{x} = -S_\theta u_3 \\ m\ddot{y} = C_\theta S_\phi u_3 \\ m\ddot{z} = C_\theta C_\phi u_3 - mg \end{array} \tag{13}$$

## 3.4 Rotational Motion of the Model without Axes Orientation

The three inputs in torque are given by:

$$\begin{array}{l} \tau_\theta = l\,(f_2 - f_4) \\ \tau_\phi = l\,(f_1 - f_3) \\ \tau_\psi = lk\,(f_1 - f_2 + f_3 - f_4) \end{array} \tag{14}$$

The vertical controller is: $u_3 = f_1 + f_3 + f_2 + f_4$

Using the translational and rotational motions (13) and (14), equations of the dynamic are detailed by

$$\begin{array}{c} m\ddot{x} = -S_\theta u_3 \\ m\ddot{y} = C_\theta S_\phi u_3 \\ m\ddot{z} = C_\theta C_\phi u_3 - mg \\ \ddot{\theta} = \tilde{\tau}_\theta;\ \ddot{\phi} = \tilde{\tau}_\phi;\ \ddot{\psi} = \tilde{\tau}_\psi \end{array} \tag{15}$$

***Remark:*** As shown in the system (2), the three inputs torque see the equation (7), the yaw $\tau_\psi$ is equal to zero if we take $\xi_1 = \xi_3 = 0$. Then, with the proposed sense of rotations (see figure 3), we can not generate yaw motions if rotors 1 and 3 are not oriented. With $\xi_1 = \xi_3 = 0$, to obtain yaw motions, the rotor sense of rotations is identical of that of the Quadrotor.
Then rotors 1 and 3 are with the same sense of rotations, while rotors 2 and 4 are in opposite sense (see figure 4).
With or without axes orientation, the rotational part can be easily linearized with static feedback control laws. Then, we get

$$\begin{array}{l} \ddot{\theta} = u_4 \\ \ddot{\phi} = u_5 \\ \ddot{\psi} = u_6 \end{array} \tag{16}$$

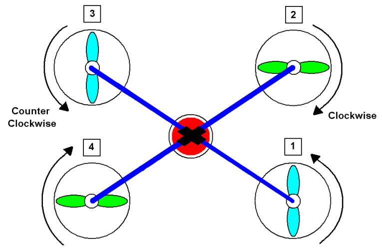


Figure 4: Rotor rotations with yaw motions.

with

$$\begin{array}{l} u_4 = \frac{1}{I_{xx} C_\phi}(\tau_\theta + I_{xx} S_\phi \dot{\phi}\dot{\theta}) \\ u_5 = \frac{1}{I_{yy} C_\theta C_\phi}(\tau_\phi + I_{yy} S_\phi C_\theta \dot{\phi}^2 + I_{yy} S_\theta C_\phi \dot{\theta}\dot{\phi}) \\ u_6 = \frac{1}{I_{zz}} \tau_\psi \end{array} \tag{17}$$

# 4 BACKSTEPPING BASED CONTROLLER

Backstepping controllers are especially useful when some states are controlled through other states. As it was observed in the previous section, in order to control the $x$ and $y$ motion of the X4-flyer, tilt angles need to be controlled. Therefore a backstepping controller has been developed in this section. Similar ideas of using backstepping with visual serving have been developed for a traditional helicopter by Hamel and Mahony (Hamel and Mahony, 2000). As well as the backstepping controllers was applied for Quadrotor by Altug et al (Altug, 2003) (Altug et al., 2003) (Altug et al., 2002).

## 4.1 ”Backstepping” Application to the Model without Axes Orientation

### 4.1.1 Altitude and yaw control

The altitude and the yaw on the other hand, can be controlled by a PD controller. With through the equation of the following movement $(z)$.

$$m\ddot{z} = C_\theta C_\phi u_3 - mg \tag{18}$$

The control of the vertical position (altitude) can be obtained considering the following control input

$$u_3 = m\left(g + \ddot{z}_r - k_z^1\,(\dot{z} - \dot{z}_r) - k_z^2\,(z - z_r)\right) \tag{19}$$

with

$$\ddot{z} = \ddot{z}_r - k_z^1\,(\dot{z} - \dot{z}_r) - k_z^2\,(z - z_r) \tag{20}$$

$z_r$ is the desired altitude. The yaw attitude can be stabilized to a desired value with the following tracking feedback control

$$u_6 = \ddot{\psi}_r - k_\psi^1(\dot{\psi} - \dot{\psi}_r) - k_\psi^2(\psi - \psi_r) \tag{21}$$

where $k_z^1$, $k_z^2$, $k_\psi^1$, $k_\psi^2$ are the coefficients of stable polynomial.

#### 4.1.2 Roll control $(\phi, y)$

First we notice that motion in the $y$ direction can be controlled through the changes of the roll angle. These variables are related by the cascade system

$$\begin{cases} m\ddot{y} = C_\theta S_\phi u_3 \\ \ddot{\phi} = u_5 \end{cases} \tag{22}$$

This leads to a backstepping controller for $y - \phi$ control given by

$$u_5 = \frac{1}{u_3 C_\theta C_\phi}(5y + 10\dot{y} + u_3 \Theta_{\theta,\phi}) \tag{23}$$

where

$$\Theta_{\theta,\phi} = \begin{pmatrix} 9S_\phi C_\theta + 4\dot{\phi} C_\phi C_\theta - \dot{\phi}^2 S_\phi C_\theta - \\ 2\dot{\theta} S_\phi S_\theta + \dot{\phi}\dot{\theta} C_\phi S_\theta - \dot{\theta}\dot{\phi} C_\phi S_\theta + \dot{\theta}^2 S_\phi C_\theta \end{pmatrix} \tag{24}$$

#### 4.1.3 Pitch control $(\theta, x)$

To develop a controller for motion along the $x$ axis, similar analysis is needed. The equation of motion of the X4-flyer on $x$ is given as

$$\begin{cases} m\ddot{x} = -S_\theta u_3 \\ \ddot{\theta} = u_4 \end{cases} \tag{25}$$

This leads to a backstepping controller for $x - \theta$ control given by

$$u_4 = \frac{1}{u_3 C_\theta}(-5x - 10\dot{x} + u_3 \Theta_\theta) \tag{26}$$

where

$$\Theta_\theta = 9S_\theta + 4\dot{\theta} C_\theta - \dot{\theta}^2 S_\theta \tag{27}$$

## 4.2 Model with Axes Orientation

#### 4.2.1 Control input for $(z - y)$ motions

We propose to control motion along $y$ and $z$ directions through $u_3$ and $u_2$, respectively. So we have the proposition (28).

$$\begin{pmatrix} \ddot{y} \\ \ddot{z} \end{pmatrix} = \frac{1}{m} H \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} - \begin{pmatrix} 0 \\ g \end{pmatrix} \tag{28}$$

where

$$H = \begin{pmatrix} S_\psi S_\theta S_\phi + C_\psi C_\phi & C_\theta S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi & C_\theta C_\phi \end{pmatrix} \tag{29}$$

For the given conditions in $\psi$ and $\theta$ , the 2 by 2 matrix (29) is invertible. Then a nonlinear decoupling feedback permits to write the following decoupled linear dynamics

$$\begin{matrix} \ddot{y} = \nu_y \\ \ddot{z} = \nu_z \end{matrix} \tag{30}$$

Then we can deduce from (30) the linear controller

$$\begin{matrix} \nu_y = \ddot{y}_r - k_y^1(\dot{y} - \dot{y}_r) - k_r^2(y - y_r) \\ \nu_z = \ddot{z}_r - k_z^1(\dot{z} - \dot{z}_r) - k_z^2(z - z_r) \end{matrix} \tag{31}$$

With the $k_y^i$ and $k_z^i$ are the coefficients of a polynomial of Hurwitz

**Proposition:** Consider

$$(\psi, \theta) \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[ \tag{32}$$

with the controllers (33) and (34)

$$u_2 = \frac{C_\phi}{C_\psi}(m\ddot{y}) - \frac{S_\phi}{C_\psi}(m(\ddot{z} + g)) \tag{33}$$

$$\begin{aligned} u_3 = {} & \tfrac{-S_\psi S_\theta C_\phi + C_\psi S_\phi}{C_\theta C_\psi}(m\ddot{y}) + \\ & \tfrac{S_\psi S_\theta S_\phi + C_\psi C_\phi}{C_\theta C_\psi}(m(\ddot{z} + g)) \end{aligned} \tag{34}$$

The dynamic of $y$ and $z$ are linearly decoupled and exponentially-asymptotically stable with the appropriate choice of the gain controller parameters.

#### 4.2.2 Control input for the $x$ motion

To control the movement along the $x$ axis, the backstepping controller is used. The noted controller $x - \theta$ is given by the equation (35):

$$\begin{cases} m\ddot{x} = S_\psi C_\theta u_2 - S_\theta u_3 \\ \ddot{\theta} = u_4 \end{cases} \tag{35}$$

One supposes it exists a time $T_f^1$ such that ∀t∈ $\left[T_0, T_f^1\right]$, $u_3 > 0$, then the dynamic of $x$ is decoupled under the following controller

$$u_4 = \frac{1}{u_3 C_\theta + u_2 S_\theta S_\psi}(-5x - 10\dot{x} + u_3\Theta_\theta + u_2\Theta_{\theta,\psi}) \tag{36}$$

where

$$\Theta_\theta = 9S_\theta + 4\dot{\theta} C_\theta - \dot{\theta}^2 S_\theta \tag{37}$$

and

$$\Theta_{\theta,\psi} = \begin{pmatrix} 9S_\psi C_\theta + 4\dot{\theta} C_\psi C_\theta - \dot{\psi}^2 S_\psi C_\theta - \\ 2\dot{\psi} S_\psi S_\theta + \dot{\psi}\dot{\theta} C_\psi S_\theta - \dot{\theta}\dot{\psi} C_\psi S_\theta + \dot{\theta}^2 S_\psi C_\theta \end{pmatrix} \tag{38}$$

# 5 TRAJECTORY GENERATION AND POINT TO POINT STEERING

Due to the structure limit of the X4-flyer, motion can be asserted only in straight line along the $x, y$ and $z$ directions. In our case, that is sufficient to navigate in a region. Otherwise, an other version of the engine is under study by the group. The version flyer is to make easy manoeuvres in corners with arc of circle. In the following, we solve the tracking problem as point to point steering one over a finite interval of time. Then we take each ending point with its final time as a new starting point.

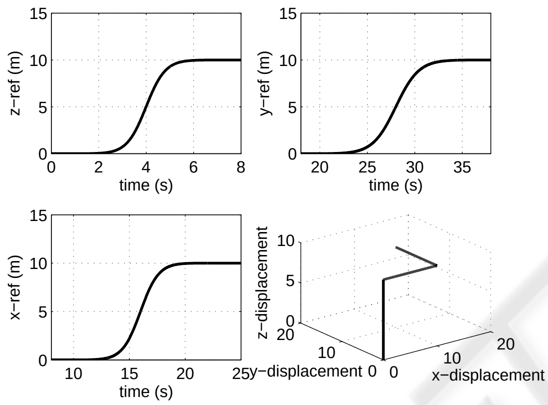


Figure 5: Motion planning with $h_d = 10m$.

Figure 5 illustrate the reference trajectory along the $x, y$ and $z$ directions. As we see, the X4-flyer will fly in the $z$ direction followed by the $x-motion$ and the $y-motion$. The reference trajectory is parameterized as

$$z^r(t) = h_d \frac{t^5}{t^5 + (T_f^1 - t)^5} \tag{39}$$

where $h_d$ is the desired altitude and $(T_f^1)$ the final time. In order to solve the point to point steering control, the end point of the trajectory (39) can be adopted as initial point to move along $x$, then we have

$$x^r(t) = h_d \frac{(t - T_f^1)^5}{(t - T_f^1)^5 + (T_f^2 - (t - T_f^1))^5} \tag{40}$$

As soon as for $y^r\,(t)$

$$y^r(t) = h_d \frac{(t - T_f^2)^5}{(t - T_f^2)^5 + (T_f^3 - (t - T_f^2))^5} \tag{41}$$

The constraints to perform these trajectories are such that

$$\begin{cases} z^r(0) = x^r(T_f^1) = y^r(T_f^2) = 0 \\ z^r(T_f^1) = x^r(T_f^2) = y^r(T_f^3) = h_d \\ \dot{z}^r(0) = \dot{x}^r(T_f^1) = \dot{y}^r(T_f^2) = 0 \\ \dot{z}^r(T_f^1) = \dot{x}^r(T_f^2) = \dot{y}^r(T_f^3) = 0 \\ \ddot{z}^r(0) = \ddot{x}^r(T_f^1) = \ddot{y}^r(T_f^2) = 0 \\ \ddot{z}^r(T_f^1) = \ddot{x}^r(T_f^2) = \ddot{y}^r(T_f^3) = 0 \end{cases} \tag{42}$$

Minimizing the time of displacement implies that the X4-flyer accelerates at the beginning and decelerates at the arrival.

# 6 SIMULATION RESULTS

Two engine models were studied and controlled using the backstepping technique which $(a)$ present the model with axes orientation and $(b)$ the model without axes orientation.

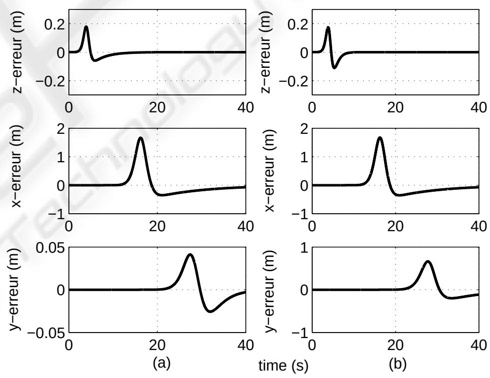


Figure 6: Displacement errors: (a) with axes orientation - (b) without axes orientation.

Figure 6 show displacement errors according to all the directions for the models with and without axes orientations. It is noticed that the error thus tends to zero towards the desired positions.

Figure 7, we notices that the angles $\theta$ and $\phi$ control the engine for displacements along the axes $x$ and $y$. These angles tend to zero value. It is also shown in figure 8$(a)$ that we can stabilize the system to make a following movement by the swivelling of the engine actuators 1 and 3.

According to the figure 8, which represent our vehicle input, we remark that the input $u_3 = mg$ at the equilibrium state is always verified. The inputs $u_2$, $u_4$ and $u_5$ tend to zero after having carried out the desired orientation of the vehicle. These Figure 8 also show the effectiveness of the used controllers laws.

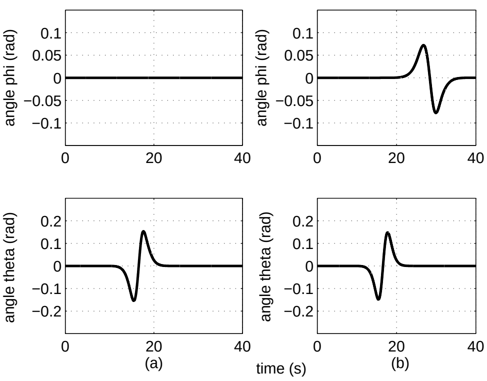


Figure 7: The pitch $\theta$ and the roll $\phi$: (a) with axes orientation - (b) without axes orientation.

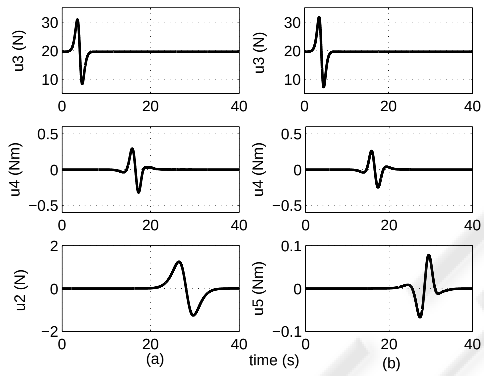


Figure 8: Inputs $u_2,\ u_3,\ u_4\ and\ u_5$ for the $xyz$ displacement: (a) with axes orientation - (b) without axes orientation.

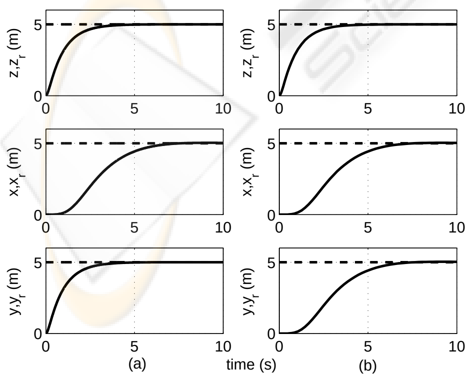


Figure 9: Without motion planning with $h_d = 5m$: (a) with axes orientation - (b) without axes orientation.

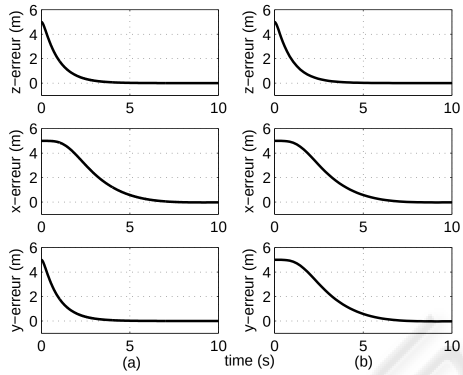


Figure 10: Tracking errors without motion planning ($z_r = x_r = y_r = 5m$): (a) with axes orientation - (b) without axes orientation.

Figures 9,10,11 and 12, show the system without motion planning. Motion in different directions $z$, $x$ and $y$ is also tested and shown by figure 9. In addition we show that the behavior of errors, given by figure 10 is verified. At the equilibrium, attitudes of $\theta$ and $\phi$ are equal to zero (figure 11).
Without motion planning, the amplitude of controllers is important (figure 12) and a maximum of energy is asserted which is requested for flying vehicles.

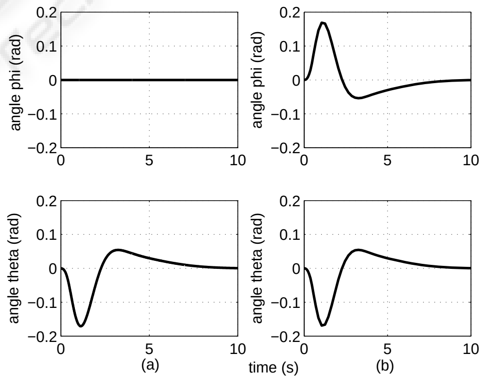


Figure 11: The pitch $\theta$ and the roll $\phi$ for the vehicle without motion planning: (a) with axes orientation - (b) without axes orientation.

## 7 CONCLUSION

The study of the stabilization with and without a predefined trajectory of the mini-flying robot with four rotors (X4-flyer) was discussed in this paper. The importance of the trajectory generation and its consequences with respect to amplitude of the used controller, was highlited. With the proposed motion plan-

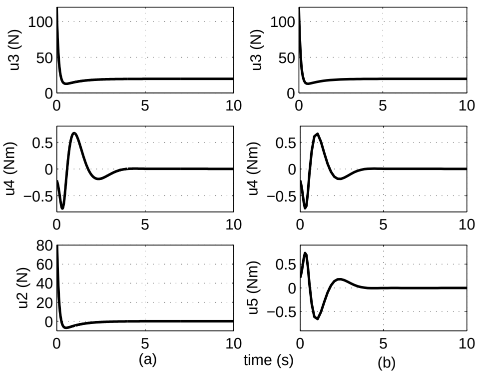


Figure 12: Inputs $u_2,\ u_3,\ u_4\ and\ u_5$ for the vehicle without motion planning: (a) with axes orientation - (b) without axes orientation.

ning, actuator saturations can be overcomed. Consequently, economy in energy of batteries can be asserted during the fly. The backstepping technique was successfully applied and enabled us to design control algorithms ensuring the vehicle displacement from an initial position to a desired position. The backstepping approach used requires the well knowledge of the system model and parameters. Future work is to develop the fuzzy controller based algorithm (does not require the good knowledge of the model) (Maaref and Barret, 2001) and to make the comparison of both controllers. A realization of a control system based on engine sensors information is envisaged.

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