ROBUST FUZZY CONTROLLER DESIGN FOR UNCERTAIN DESCRIPTOR MARKOVIAN JUMP SYSTEMS

Wudhichai Assawinchaichote
Department of Electronic and Telecommunication Engineering
King Mongkut's University of Technology Thonburi, 91 Prachautits Rd., Bangkok 10140, Thailand

Sing Kiong Nguang
Department of Electrical and Computer Engineering
The University of Auckland, Private Bag 92019 Auckland, New Zealand

Keywords: TS Fuzzy model, $\mathcal{H}_\infty$ control, Markovian jumps, LMI.

Abstract: This paper examines the problem of designing a robust $\mathcal{H}_\infty$ state-feedback controller for a class of uncertain nonlinear descriptor Markovian jump systems described by a Takagi-Sugeno (TS) fuzzy model with Markovian jumps. Based on a linear matrix inequality (LMI) approach, LMI-based sufficient conditions for the uncertain nonlinear descriptor Markovian jump systems to have an $\mathcal{H}_\infty$ performance are derived. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard nonlinear descriptor systems. A numerical example is provided to illustrate the design developed in this paper.

1 INTRODUCTION

Markovian jump systems, sometimes called hybrid systems with a state vector, consists of two components; i.e., the state (differential equation) and the mode (Markov process). The Markovian jump system changes abruptly from one mode to another mode caused by some phenomenon such as environmental disturbances, changing subsystem interconnections and fast variations in the operating point of the system plant, etc. The switching between modes is governed by a Markov process with the discrete and finite state space. Over the past few decades, the Markovian jump systems have been extensively studied by many researchers; see (Kushner, 1967; Dynkin, 1965; Wonham, 1968; X. Feng and Chizeck, 1992; de Souza and Fragoso, 1993; Boukas and Liu, 2001; Boukas and Yang, 1999; Rami and Ghaoui, 1995; Shi and Boukas, 1997). This is due to the fact that jumping systems have been a subject of the great practical importance.

For the past three decades, descriptor systems or called singularly perturbed systems have been intensively studied by many researchers; see (Shi and Boukas, 1997; K. Benjelloun and Costa, 1997; E. K. Boukas and Liu, 2001; V. Dragan and Boukas, 1999; Pan and Basar, 1993; Pan and Basar, 1994; Fridman, 2001; Shi and Dragan, 1999; P. V. Kokotovic and O'Reilly, 1986). Singularly perturbed systems also known as multiple time-scale dynamic systems normally occur due to the presence of small "parasitic" parameters, typically small time constants, masses, etc. In state space, such systems are commonly modelled using the mathematical framework of singular perturbations, with a small parameter, say ε, determining the degree of separation between the "slow" and "fast" modes of the system. However, it is necessary to note that it is possible to solve the singularly perturbed systems without separating between slow and fast mode subsystems. But the requirement is that the "parasitic" parameters must be large enough. In the case of having very small "parasitic" parameters which normally occur in the description of various physical phenomena, a popular approach adopted to handle these systems is based on the so-called reduction technique. According to this technique the fast variables are replaced by their steady states obtained with "frozen" slow variables and controls, and the slow dynamics is approximated by the corresponding reduced order system. This time-scale is asymptotic, that is, exact in the limit, as the ratio of the speeds of the slow versus the fast dynamics tends to zero.

In the last few years, the research on singularly perturbed systems in the $\mathcal{H}_\infty$ sense has been highly recognized in control area due to the great practical importance. $\mathcal{H}_\infty$-optimal control of singularly perturbed linear systems under either perfect state measurements or imperfect state measurements has been

Assawinchaichote W. and Kiong Nguang S. (2005).
ROBUST FUZZY CONTROLLER DESIGN FOR UNCERTAIN DESCRIPTOR MARKOVIAN JUMP SYSTEMS.
In *Proceedings of the Second International Conference on Informatics in Control, Automation and Robotics*, pages 91-97
DOI: 10.5220/0001156800910097

investigated via differential game theoretics approach. Although many researchers have studied the $\mathcal{H}_\infty$ control design of linear singularly perturbed systems for many years, the $\mathcal{H}_\infty$ control design of nonlinear singularly perturbed systems remains as an open research area. This is due to, in general, nonlinear singularly perturbed systems can not be decomposed into slow and fast subsystems.

Recently, a great amount of effort has been made on the design of fuzzy $\mathcal{H}_\infty$ for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model; see (Nguang and Shi, 2001; Han and Feng, 1998; B. S. Chen and He, 2001; K. Tanaka and Wang, 1996). Recent studies (Nguang and Shi, 2001; Han and Feng, 1998; B. S. Chen and He, 2001; K. Tanaka and Wang, 1996; H. O. Wang and Griffin, 1996) show that a fuzzy model can be used to approximate global behaviors of a highly complex nonlinear system. In this fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling which uses a single model to describe the global behavior of a system, fuzzy modelling is essentially a multi-model approach in which simple submodels (linear models) are combined to describe the global behavior of the system. Employing the existing fuzzy results (Nguang and Shi, 2001; Han and Feng, 1998; B. S. Chen and He, 2001; K. Tanaka and Wang, 1996; H. O. Wang and Griffin, 1996) on the singularly perturbed system, one ends up with a family of ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes. In general, ill-conditioned linear matrix inequalities are very difficult to solve.

What we intend to do in this paper is to design a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of uncertain nonlinear singularly perturbed systems with Markovian jumps. First, we approximate this class of uncertain nonlinear singularly perturbed systems with Markovian jumps by a Takagi-Sugeno fuzzy model with Markovian jumps. Then based on an LMI approach, we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value. To alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, these ill-conditioned LMIs are decomposed into ε-independent LMIs and ε-dependent LMIs. The ε-independent LMIs are not ill-conditioned and the ε-dependent LMIs tend to zero when ε approaches to zero. If ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable. The proposed approach does not involve the separation of states into slow and fast ones, and it can be applied not only to standard, but also to nonstandard singularly perturbed systems.

This paper is organized as follows. In Section 2, system descriptions and definition are presented. In Section 3, based on an LMI approach, we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value for the system described in Section 2. The validity of this approach is demonstrated by an example from a literature in Section 4. Finally, conclusions are given in Section 5.

2 SYSTEM DESCRIPTIONS AND DEFINITIONS

The class of nonlinear uncertain singularly perturbed system with Markovian jumps under consideration is described by the following TS fuzzy model with Markovian jumps:

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\nu(t))\times \\ &\quad \Big[[A_i(\eta(t)) + \Delta A_i(\eta(t))]x(t) \\ &\quad +[B_{1_i}(\eta(t)) + \Delta B_{1_i}(\eta(t))]w(t) \\ &\quad +[B_{2_i}(\eta(t)) + \Delta B_{2_i}(\eta(t))]u(t)\Big], \\ z(t) &= \sum_{i=1}^{r} \mu_i(\nu(t))\times \\ &\quad \Big[[C_{1_i}(\eta(t)) + \Delta C_{1_i}(\eta(t))]x(t) \\ &\quad +[D_{12_i}(\eta(t)) + \Delta D_{12_i}(\eta(t))]u(t)\Big] \end{aligned} \tag{1}$$

where $E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$, $\varepsilon > 0$ is the singular perturbation parameter, $\nu(t) = [\nu_1(t) \ \cdots \ \nu_\vartheta(t)]$ is the premise variable that may depend on states in many cases, $\mu_i(\nu(t))$ denote the normalized time-varying fuzzy weighting functions for each rule, ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, $z(t) \in \Re^s$ is the controlled output, the matrix functions $A_i(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $C_{1_i}(\eta(t))$, $D_{12_i}(\eta(t))$, $\Delta A_i(\eta(t))$, $\Delta B_{1_i}(\eta(t))$, $\Delta B_{2_i}(\eta(t))$, $\Delta C_{1_i}(\eta(t))$ and $\Delta D_{12_i}(\eta(t))$ are of appropriate dimensions. $\{\eta(t))\}$ is a continuous-time discrete-state Markov process taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, s\}$ with transition probability matrix $Pr \stackrel{\Delta}{=} \{P_{\imath k}(t)\}$ given by

$$\begin{aligned} P_{\imath k}(t) &= Pr(\eta(t+\Delta) = k|\eta(t) = \imath) \\ &= \begin{cases} \lambda_{\imath k}\Delta + O(\Delta) & \text{if } \imath \neq k \\ 1 + \lambda_{\imath\imath}\Delta + O(\Delta) & \text{if } \imath = k \end{cases} \end{aligned} \tag{2}$$

where $\Delta > 0$, and $\lim_{\Delta \longrightarrow 0} \frac{O(\Delta)}{\Delta} = 0$. Here $\lambda_{\imath k} \geq 0$ is the transition rate from mode $\imath$ (system operating mode) to mode k ($\imath \neq k$), and

$$\lambda_{\imath\imath} = -\sum_{k=1, k\neq \imath}^{s} \lambda_{\imath k}. \qquad (3)$$

For the convenience of notations, we let $\mu_i \triangleq \mu_i(\nu(t))$, $\eta = \eta(t)$, and any matrix $M(\mu, \imath) \triangleq M(\mu, \eta = \imath)$. The matrix functions $\Delta A_i(\eta)$, $\Delta B_{1_i}(\eta)$, $\Delta B_{2_i}(\eta)$, $\Delta C_{1_i}(\eta)$ and $\Delta D_{12_i}(\eta)$ represent the time-varying uncertainties in the system and satisfy the following assumption.

Assumption 1

$$\Delta A_i(\eta) = F(x(t), \eta, t) H_{1_i}(\eta),$$
$$\Delta B_{1_i}(\eta) = F(x(t), \eta, t) H_{2_i}(\eta),$$
$$\Delta B_{2_i}(\eta) = F(x(t), \eta, t) H_{3_i}(\eta),$$
$$\Delta C_{1_i}(\eta) = F(x(t), \eta, t) H_{4_i}(\eta),$$
$$\text{and} \ \ \Delta D_{12_i}(\eta) = F(x(t), \eta, t) H_{5_i}(\eta),$$

where $H_{j_i}(\eta)$, $j = 1, 2, \cdots, 5$ are known matrices which characterize the structure of the uncertainties. Furthermore, there exists a positive function $\rho(\eta)$ such that the following inequality holds:

$$\| F(x(t), \eta, t) \| \leq \rho(\eta). \qquad (4)$$

We recall the following definition.

Definition 1 *Suppose γ is a given positive number. A system of the form (1) is said to have the $\mathcal{L}_2$-gain less than or equal to γ if*

$$\mathbf{E}\left[\int_0^{T_f} \left\{ z^T(t) z(t) - \gamma^2 w^T(t) w(t) \right\} dt \right] \leq 0, \quad (5)$$

where $x(0) = 0$ and $\mathbf{E}\left[\cdot\right]$ stands for the mathematical expectation, for all T_f and all $w(t) \in \mathcal{L}_2[0, T_f]$.

Note that for the symmetric block matrices, we use $(*)$ as an ellipsis for terms that are induced by symmetry.

3 ROBUST $\mathcal{H}_\infty$ FUZZY STATE-FEEDBACK CONTROL DESIGN

This section provides the LMI-based solutions to the problem of designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value.

First, we consider the following $\mathcal{H}_\infty$ fuzzy state-feedback which is inferred as the weighted average of the local models of the form:

$$u(t) = \sum_{j=1}^{r} \mu_j K_j(\imath) x(t). \qquad (6)$$

Then, we describe the problem under our study as follows.

Problem Formulation: Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller of the form (6) such that the inequality (5) holds.

Before presenting our first main result, we recall the following lemma.

Lemma 1 *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, for $\imath = 1, 2, \cdots, s$, if there exist matrices $P_\varepsilon(\imath) = P_\varepsilon^T(\imath)$, positive constants $\delta(\imath)$ and matrices $Y_j(\imath)$, $j = 1, 2, \cdots, r$ such that the following ε-dependent linear matrix inequalities hold:*

$$\begin{aligned} P_\varepsilon(\imath) &> 0 & & (7)\\ \Psi_{ii}(\imath, \varepsilon) &< 0, \quad i = 1, 2, \cdots, r & & (8)\\ \Psi_{ij}(\imath, \varepsilon) + \Psi_{ji}(\imath, \varepsilon) &< 0, \quad i < j \leq r & & (9)\end{aligned}$$

where

$$\Psi_{ij}(\imath, \varepsilon) = \begin{pmatrix} \Phi_{ij}(\imath, \varepsilon) & (*)^T & (*)^T & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) & -\gamma\mathcal{R}(\imath) & (*)^T & (*)^T \\ \Upsilon_{ij}(\imath, \varepsilon) & 0 & -\gamma\mathcal{R}(\imath) & (*)^T \\ \mathcal{Z}^T(\imath, \varepsilon) & 0 & 0 & -\mathcal{P}(\imath, \varepsilon) \end{pmatrix}$$

$$\begin{aligned}\Phi_{ij}(\imath, \varepsilon) &= A_i(\imath) E_\varepsilon^{-1} P_\varepsilon(\imath) + E_\varepsilon^{-1} P_\varepsilon(\imath) A_i^T(\imath) \\ &\quad + B_{2_i}(\imath) Y_j(\imath) + Y_j^T(\imath) B_{2_i}^T(\imath) \\ &\quad + \lambda_{\imath\imath} E_\varepsilon^{-1} P_\varepsilon(\imath), \\ \Upsilon_{ij}(\imath, \varepsilon) &= \tilde{C}_{1_i}(\imath) E_\varepsilon^{-1} P_\varepsilon(\imath) + \tilde{D}_{12_i}(\imath) Y_j(\imath), \\ \mathcal{R}(\imath) &= diag\left\{\delta(\imath) I, I, \delta(\imath) I, I\right\}, \\ \mathcal{Z}(\imath, \varepsilon) &= \Big(\sqrt{\lambda_{\imath 1}} E_\varepsilon^{-1} P_\varepsilon(\imath) \ \cdots \ \sqrt{\lambda_{\imath(\imath-1)}} E_\varepsilon^{-1} P_\varepsilon(\imath) \\ &\quad \sqrt{\lambda_{\imath(\imath+1)}} E_\varepsilon^{-1} P_\varepsilon(\imath) \ \cdots \ \sqrt{\lambda_{\imath s}} E_\varepsilon^{-1} P_\varepsilon(\imath) \Big), \\ \mathcal{P}(\imath, \varepsilon) &= diag\Big\{ E_\varepsilon^{-1} P_\varepsilon(1), \cdots, E_\varepsilon^{-1} P_\varepsilon(\imath - 1), \\ &\quad E_\varepsilon^{-1} P_\varepsilon(\imath + 1), \cdots, E_\varepsilon^{-1} P_\varepsilon(s) \Big\},\end{aligned}$$

with

$$\tilde{B}_{1_i}(\imath) = [\ I \quad I \quad I \quad B_{1_i}(\imath)\ ]$$

$$\tilde{C}_{1_i}(\imath) = \Big[\gamma\rho(\imath)H_{1_i}^T(\imath) \ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath) \quad 0 \ \ \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\Big]^T$$

$$\tilde{D}_{12_i}(\imath) = \Big[0 \ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{5_i}^T(\imath) \quad \gamma\rho(\imath)H_{3_i}^T(\imath) \ \ \sqrt{2}\aleph(\imath)D_{12_i}^T(\imath)\Big]^T$$

$$\aleph(\imath) = \left(I + \rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\Big[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\|\Big]\right)^{\frac{1}{2}}$$

then the inequality (5) holds. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r}\mu_j K_{\varepsilon_j}(\imath)x(t) \tag{10}$$

where

$$K_{\varepsilon_j}(\imath) = Y_j(\imath)(P_\varepsilon(\imath))^{-1}E_\varepsilon. \tag{11}$$

Proof: The desired result can be carried out by a similar technique used in (D. P. de Farias and Costa, 2000), (Nguang and Shi, 2003), and (Nguang and Shi, 2001). Due to limited pages, the detail of the proof is omitted for brevity. ■

Remark 1 *The linear matrix inequalities given in Lemma 1 becomes ill-conditioned when ε is sufficiently small, which is always the case for the singularly perturbed system. In general, these ill-conditioned linear matrix inequalities are very difficult to solve. Thus, to alleviate these ill-conditioned linear matrix inequalities, we have the following theorem which does not depend on ε.* □

Now we are in the position to present our first result.

Theorem 1 *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, for $\imath = 1, 2, \cdots, s$, if there exist matrices $P(\imath)$, positive constants $\delta(\imath)$ and matrices $Y_j(\imath)$, $j = 1, 2, \cdots, r$ such that the following ε-independent linear matrix inequalities hold:*

$$EP(\imath) + P(\imath)D > 0 \tag{12}$$

$$\Psi_{ii}(\imath) < 0, \quad i = 1, 2, \cdots, r \tag{13}$$

$$\Psi_{ij}(\imath) + \Psi_{ji}(\imath) < 0, \quad i < j \le r \tag{14}$$

where $EP(\imath) = P^T(\imath)E$, $P(\imath)D = DP^T(\imath)$, $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$,

$$\Psi_{ij}(\imath) = \begin{pmatrix} \Phi_{ij}(\imath) & (*)^T & (*)^T & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) & -\gamma\mathcal{R}(\imath) & (*)^T & (*)^T \\ \Upsilon_{ij}(\imath) & 0 & -\gamma\mathcal{R}(\imath) & (*)^T \\ \mathcal{Z}^T(\imath) & 0 & 0 & -\mathcal{P}(\imath) \end{pmatrix}$$

$$\begin{aligned}
\Phi_{ij}(\imath) &= A_i(\imath)P(\imath) + P^T(\imath)A_i^T(\imath) + B_{2_i}(\imath)Y_j(\imath) + Y_j^T(\imath)B_{2_i}^T(\imath) + \lambda_{\imath\imath}\tilde{\tilde{P}}(\imath),\\
\Upsilon_{ij}(\imath) &= \tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath),\\
\mathcal{R}(\imath) &= \mathit{diag}\left\{\delta(\imath)I, I, \delta(\imath)I, I\right\},\\
\mathcal{Z}(\imath) &= \Big(\sqrt{\lambda_{\imath 1}}\tilde{\tilde{P}}(\imath) \ \cdots \ \sqrt{\lambda_{\imath(\imath-1)}}\tilde{\tilde{P}}(\imath) \ \sqrt{\lambda_{\imath(\imath+1)}}\tilde{\tilde{P}}(\imath) \ \cdots \ \sqrt{\lambda_{\imath s}}\tilde{\tilde{P}}(\imath)\Big),\\
\mathcal{P}(\imath) &= \mathit{diag}\Big\{\tilde{\tilde{P}}(1), \cdots, \tilde{\tilde{P}}(\imath-1), \tilde{\tilde{P}}(\imath+1), \cdots, \tilde{\tilde{P}}(s)\Big\},\\
\tilde{\tilde{P}}(\imath) &= \frac{P(\imath) + P^T(\imath)}{2}
\end{aligned}$$

with

$$\tilde{B}_{1_i}(\imath) = [\ I \quad I \quad I \quad B_{1_i}(\imath)\ ]$$

$$\tilde{C}_{1_i}(\imath) = \Big[\gamma\rho(\imath)H_{1_i}^T(\imath) \ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath) \quad 0 \ \ \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\Big]^T$$

$$\tilde{D}_{12_i}(\imath) = \Big[0 \ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{5_i}^T(\imath) \quad \gamma\rho(\imath)H_{3_i}^T(\imath) \ \ \sqrt{2}\aleph(\imath)D_{12_i}^T(\imath)\Big]^T$$

$$\aleph(\imath) = \left(I + \rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\Big[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\|\Big]\right)^{\frac{1}{2}}$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ such that the inequality (5) holds for $\varepsilon \in (0, \hat{\varepsilon}]$. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{i=1}^{r}\mu_j K_j(\imath)x(t) \tag{15}$$

where

$$K_j(\imath) = Y_j(\imath)(P(\imath))^{-1}. \tag{16}$$

Proof: Due to limited pages, the detail of the proof is omitted for brevity. ■

4 ILLUSTRATIVE EXAMPLE

Consider a modified series dc motor model based on (Mehta and Chiasson, 1998) as shown in Fig. 1 which is governed by the following difference equations:

$$\begin{aligned}
J\frac{d\tilde{\omega}(t)}{dt} &= K_m L_f \tilde{i}^2(t) - (D + \Delta D)\tilde{\omega}(t)\\
L\frac{d\tilde{i}(t)}{dt} &= -R\tilde{i}(t) - K_m L_f \tilde{i}(t)\tilde{\omega}(t) + \tilde{V}(t)
\end{aligned} \tag{17}$$

where $\tilde{\omega}(t) = \omega(t) - \omega_{ref}(t)$ is the deviation of the actual angular velocity from the desired angular velocity, $\tilde{i}(t) = i(t) - i_{ref}(t)$ is the deviation of the actual current from the desired current, $\tilde{V}(t) = V(t) - V_{ref}(t)$ is the deviation of the actual input voltage from the desired input voltage, J is the moment of inertia, K_m is the torque/back emf constant, D is the viscous friction coefficient, and R_a, R_f, L_a and L_f are the armature resistance, the field winding resistance, the armature inductance and the field winding inductance, respectively, with $R \triangleq R_f + R_a$ and $L \triangleq L_f + L_a$. Note that in a typical series-connected dc motor, the condition $L_f \gg L_a$ holds. When one obtains a series-connected dc motor, we have $i(t) = i_a(t) = i_f(t)$. Now let us assume that $|\Delta J| \leq 0.1J$.

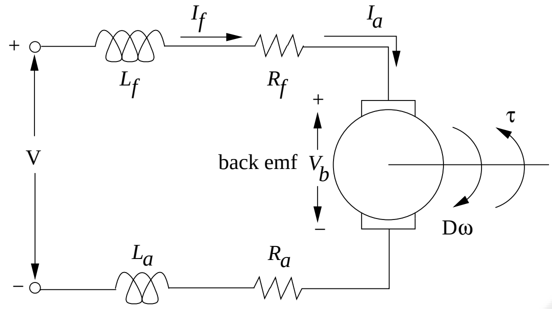

Figure 1: A modified series dc motor equivalent circuit.

Giving $x_1(t) = \tilde{\omega}(t)$, $x_2(t) = \tilde{i}(t)$ and $u(t) = \tilde{V}(t)$, (17) becomes

$$\begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{D}{(J+\Delta J)} & \frac{K_m L_f}{(J+\Delta J)}x_2(t) \\ -K_m L_f x_2(t) & -R \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{18}$$

where $\varepsilon = L$ represents a small parasitic parameter. Assume that, the system is aggregated into 3 modes as shown in Table 1:

Table 1: System Terminology.

Mode $\imath$	Moment of Inertia	$J(\imath) \pm \Delta J(\imath)$ (kg·m^2)
1	Small	0.0005 $\pm 10\%$
2	Normal	0.005 $\pm 10\%$
3	Large	0.05 $\pm 10\%$

The transition probability matrix that relates the three operation modes is given as follows:

$$P_{\imath k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

The parameters for the system are given as $R = 10\ \Omega$, $L_f = 0.005$ H, $D = 0.05$ N·m/rad/s and $K_m = 1$ N·m/A. Substituting the parameters into (18), we get

$$\begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.005}{J(\imath)}x_2(t) \\ -0.005x_2(t) & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} -\frac{0.05}{\Delta J(\imath)} & \frac{0.005}{\Delta J(\imath)}x_2(t) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$z(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

where $x(t) = [x_1^T(t) \quad x_2^T(t)]^T$ is the state variables, $w(t) = [w_1^T(t) \quad w_2^T(t)]^T$ is the disturbance input, $u(t)$ is the controlled input and $z(t)$ is the controlled output.

The control objective is to control the state variable $x_2(t)$ for the range $x_2(t) \in [N_1 \ N_2]$. For the sake of simplicity, we will use as few rules as possible. Note that Fig. 2 shows the plot of the membership function represented by

$$M_1(x_2(t)) = \frac{-x_2(t) + N_2}{N_2 - N_1}$$

$$\text{and} \quad M_2(x_2(t)) = \frac{x_2(t) - N_1}{N_2 - N_1}.$$

Knowing that $x_2(t) \in [N_1 \ N_2]$, the nonlinear system

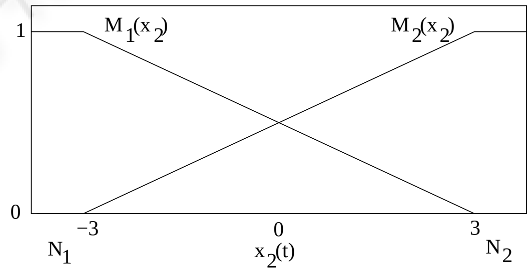

Figure 2: Membership functions for the two fuzzy set.

(19) can be approximated by the following TS fuzzy model

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \sum_{i=1}^{r} \mu_i \Big[[A_i(\imath) + \Delta A_i(\imath)]x(t) \\ &\quad + B_{1_i}(\imath)w(t) + B_{2_i}(\imath)u(t)\Big], \quad x(0) = 0, \\ z(t) &= \sum_{i=1}^{r} \mu_i \Big[C_{1_i}(\imath)x(t) + D_{12_i}(\imath)u(t)\Big], \end{aligned}$$

where μ_i is the normalized time-varying fuzzy weighting functions for each rule, $i = 1, 2$, $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $E_\varepsilon =$

$\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$, $\Delta A_1(\imath) = F(x(t),\imath,t)H_{1_1}(\imath)$, $\Delta A_2(\imath) = F(x(t),\imath,t)H_{1_2}(\imath)$,

$$A_1(1) = \begin{bmatrix} -100 & 10N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(1) = \begin{bmatrix} -100 & 10N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$A_1(2) = \begin{bmatrix} -10 & N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(2) = \begin{bmatrix} -10 & N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$A_1(3) = \begin{bmatrix} -1 & 0.1N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(3) = \begin{bmatrix} -1 & 0.1N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$B_{1_1}(\imath) = B_{1_2}(\imath) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix},$$

$$B_{2_1}(\imath) = B_{2_2}(\imath) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_{1_1}(\imath) = C_{1_2}(\imath) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\text{and } D_{12_1}(\imath) = D_{12_2}(\imath) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

with $\|F(x(t),\imath,t)\| \leq 1$. Then we have

$$H_{1_1}(\imath) = \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.05}{J(\imath)}N_1 \\ 0 & 0 \end{bmatrix}$$

$$\text{and } H_{1_2}(\imath) = \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.05}{J(\imath)}N_2 \\ 0 & 0 \end{bmatrix}.$$

In this simulation, we select $N_1 = -3$ and $N_2 = 3$. Using the LMI optimization algorithm and Theorem 1 with $\varepsilon = 0.005$, $\gamma = 1$ and $\delta(1) = \delta(2) = \delta(3) = 1$, we obtain the results given in Fig. 3 and Fig. 4.

Remark 2 *Employing results given in (Nguang and Shi, 2001; Han and Feng, 1998; B. S. Chen and He, 2001; K. Tanaka and Wang, 1996; H. O. Wang and Griffin, 1996) and Matlab LMI solver (S. Boyd and Balakrishnan, 1994), it is easy to realize that when* $\varepsilon < 0.005$ *for the state-feedback control design, LMIs become ill-conditioned and Matlab LMI solver yields an error message, "Rank Deficient". However, the state-feedback fuzzy controller proposed in this paper guarantee that the inequality (5) holds for the system (19). Fig. 3 shows the result of the changing between modes during the simulation with the initial mode at mode 1 and* $\varepsilon = 0.005$*. The disturbance input signal,* $w(t)$*, which was used during simulation is with magnitude 0.1 and frequency 1 Hz. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the* $\mathcal{H}_\infty$ *fuzzy controller is depicted in Fig. 4. The ratio of the regulated output energy to the disturbance input noise energy tends to a constant value which is about* 0.0094*. So* $\gamma = \sqrt{0.0094} = 0.0970$ *which is less than the prescribed value* 1*. Finally, Table 2 shows the performance index,* γ*, for different values of* ε*.* □

5 CONCLUSION

This paper has investigated the problem of designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of uncertainty Markovian jump nonlinear singularly perturbed systems that guarantees the $\mathcal{L}_2$-gain from an exogenous input to a regulated output to be less or equal to a prescribed value. First, we approximate this class of uncertain Markovian jump nonlinear singularly perturbed systems by a class of uncertain Takagi-Sugeno fuzzy models with Markovian jumps. Then, based on an LMI approach, LMI-based sufficient conditions for the uncertain Markovian jump nonlinear singularly perturbed systems to have an $\mathcal{H}_\infty$ performance are derived. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard nonlinear singularly perturbed systems. An illustrative example is used to illustrate the effectiveness of the proposed design techniques.

Table 2: The performance index γ for different values of ε.

	The performance index γ
ε	State-feedback control design
0.005	0.0970
0.10	0.4796
0.30	0.8660
0.40	0.9945
0.41	> 1

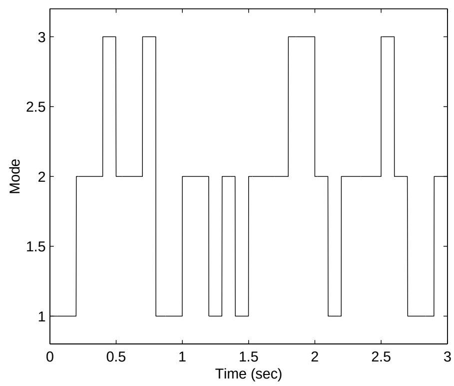

Figure 3: The result of the changing between modes during the simulation with the initial mode at mode 1.

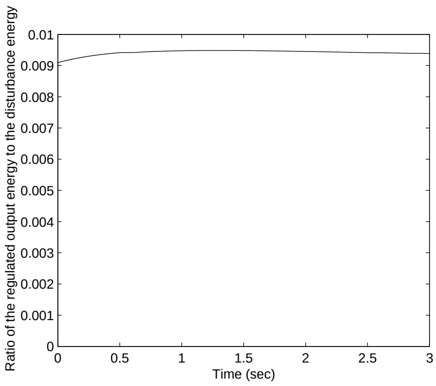

Figure 4: The ratio of the regulated output energy to the disturbance noise energy, $\left(\frac{\int_0^{T_f} z^T(t)z(t)dt}{\int_0^{T_f} w^T(t)w(t)dt}\right)$ with $\varepsilon = 0.005$.

REFERENCES

B. S. Chen, C. S. T. and He, Y. Y. (2001). Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ fuzzy output feedback control design for nonlinear dynamic systems: An lmi approach. *IEEE Trans. Fuzzy Syst.*, 8:249–265.

Boukas, E. K. and Liu, Z. K. (2001). Suboptimal design of regulators for jump linear system with time-multiplied quadratic cost. *IEEE Tran. Automat. Contr.*, 46:944–949.

Boukas, E. K. and Yang, H. (1999). Exponential stabilizability of stochastic systems with markovian jump parameters. *Automatica*, 35:1437–1441.

D. P. de Farias, J. C. Geromel, J. B. R. d. V. and Costa, O. L. V. (2000). Output feedback control of markov jump linear systems in continuous-time. *IEEE Trans. Automat. Contr.*, 45:944–949.

de Souza, C. E. and Fragoso, M. D. (1993). $\mathcal{H}_\infty$ control for linear systems with markovian jumping parameters. *Control-Theory and Advanced Tech.*, 9:457–466.

Dynkin, E. B. (1965). *Markov Process*. Springer-Verlag, Berlin.

E. K. Boukas, Z. K. L. and Liu, G. X. (2001). Delay-dependent robust stability and $\mathcal{H}_\infty$ control of jump linear systems with time-delay. *Int. J. of Contr.*

Fridman, E. (2001). State-feedback $\mathcal{H}^\infty$ control of nonlinear singularly perturbed systems. *Int. J. Robust Nonlinear Contr.*, 6:25–45.

H. O. Wang, K. T. and Griffin, M. F. (1996). An approach to fuzzy control of nonlinear systems: Stability and design issues. *IEEE Trans. Fuzzy Syst.*, 4:14–23.

Han, Z. X. and Feng, G. (1998). State-feedback $\mathcal{H}^\infty$ controller design of fuzzy dynamic system using lmi techniques. In *Fuzzy-IEEE'98*, pages 538–544.

K. Benjelloun, E. K. B. and Costa, O. L. V. (1997). $\mathcal{H}_\infty$ control for linear time delay with markovian jumping parameters. *J. of Opt. Theory and App.*, 105:73–95.

K. Tanaka, T. I. and Wang, H. O. (1996). Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stability, $\mathcal{H}_\infty$ control theory, and linear martix inequality. *IEEE Trans. Fuzzy Syst.*, 4:1–13.

Kushner, H. J. (1967). *Stochastic Stability and Control*. Academic Press, New York.

Mehta, S. and Chiasson, J. (1998). Nonlinear control of a series dc motor: Theory and experiment. *EEE Trans. Ind. Electron.*, 45:134–141.

Nguang, S. K. and Shi, P. (2001). $\mathcal{H}_\infty$ fuzzy output feedback control design for nonlinear systems: An lmi approach. In *Proc. IEEE Conf. Decision and Contr.*, pages 4352–4357.

Nguang, S. K. and Shi, P. (2003). $\mathcal{H}^\infty$ fuzzy output feedback control design for nonlinear systems: An lmi approach. *IEEE Trans. Fuzzy Syst.*, 11:331–340.

P. V. Kokotovic, H. K. K. and O'Reilly, J. (1986). *Singular Perturbation Methods in Control: Analysis and Design*. Academic Press, London.

Pan, Z. and Basar, T. (1993). $\mathcal{H}^\infty$–optimal control for singularly perturbed systems part i: Perfect state measurements. *Automatica*, 29:401–423.

Pan, Z. and Basar, T. (1994). $\mathcal{H}^\infty$–optimal control for singularly perturbed systems part ii: Imperfect state measurements. *IEEE Trans. Automat. Contr.*, 39:280–299.

Rami, M. A. and Ghaoui, L. E. (1995). $\mathcal{H}_\infty$ state-feedback control of jump linear systems. In *Proc. Conf. Decision and Contr.*, pages 951–952.

S. Boyd, L. El Ghaoui, E. F. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in Systems and Control Theory*. SIAM, Philadelphia.

Shi, P. and Boukas, E. K. (1997). $\mathcal{H}_\infty$ control for markovian jumping linear system with parametric uncertainty. *J. of Opt. Theory and App.*, 95:75–99.

Shi, P. and Dragan, V. (1999). Asymptotic $\mathcal{H}_\infty$ control of singularly perturbed system with parametric uncertainties. *Int. J. Robust NonlinearIEEE Trans. Automat. Contr.*, 44:1738–1742.

V. Dragan, P. S. and Boukas, E. K. (1999). Control of singularly perturbed system with markovian jump parameters: An $\mathcal{H}_\infty$ approach. *Automatica*, 35:1369–1378.

Wonham, W. M. (1968). On a matrix riccati equation of stochastic control. *SIAM J. Contr.*, 6:681–697.

X. Feng, K. A. Loparo, Y. J. and Chizeck, H. J. (1992). Stochastic stability properties of jump linear system. *IEEE Tran. Automat. Contr.*, 37:38–53.