DYNAMIC HYSTERESIS MODEL DERIVATED FROM LuGre MODEL

Sinuhe Benitez
CITEDI-IPN
2498 Roll Dr. 757 Otay Mesa San Diego, California, 92154 USA

Leonardo Acho and Ricardo Guerra
CITEDI-IPN
2498 Roll Dr. 757 Otay Mesa San Diego, California, 92154 USA

Keywords: Hysteresis, Dynamic Model, Friction.

Abstract: This paper presents a dynamic hysteresis model; which is a modification of the well known LuGre model. This model has been based on a modification to the LuGre model dynamic, which could be seen as a forward and backward displacement in the steady state solution of the dynamic LuGre model. The LuGre friction model is based on the average deflection of the bristles; implicitly, it is based on the relationship between stress and strain of the bristles under deformation. From the friction model point of view, this dynamic hysteresis model can capture the deformation behaviour between stress and strain beyond the elasticity region for the material (the bristles), a region where the relationship between stress and strain is no longer linear. So, our model can capture the friction phenomena of the original LuGre model and presents a new behaviour in the pre-sliding regime. Simulation results are presented to support our contribution.

1 INTRODUCTION

While working in mechanical applications, it is important to have an accurate model of the system in question. One of the more important considerations that must be taken into account in any mechanical system is the friction phenomenon. Many attempts have been made to reproduce the effects caused by friction, which is a non-linear effect. The importance of modeling friction lies in the simple fact that all mechanical systems present this phenomenon, moreover, being that it has non-linear behaviour, linear controllers have very poor performance when trying to overcome friction.

The first attempts made to model friction were static models, such as the Coulomb model and the Viscous friction model. The performance of these models was surpassed by the Dahl friction model (Dahl, 1968) and, more recently the LuGre friction model published by Carlos Canudas *et al.* (C. Canudas de Wit, 1995), which is at present one of the more widely used models for friction compensation. It has the ability to reproduce behaviours such as stick-slip motion, limit cycles, pre-sliding displacement and varying break away force. The basis for this model is an abstraction that supposes the existence of microscopic bristles on the surfaces in contact where friction is present, these bristles are elastic and hence present deformation which is captured in the model by the introduction of an internal state which represents the average deflection of said bristles. The interactions between the bristles are responsible for the effects collectively referred to as the friction phenomenon.

Research has been carried out to extend the LuGre model but it relies on the usage of memory stacks to reproduce the hysteresis phenomenon (J. Swevers, 2000). The main disadvantage of this approach is that the hysteresis behaviour is modeled by a static function.

All the friction models presented recreate the elastic behaviours of contact surfaces and the breakaway behaviours, but there is no model that can reproduce the effects that occur between the elastic region and the breakaway region, such as the segment of the stress-strain curve (Young's modulus and Hooke's law) that corresponds to non-elastic deformation region.

This paper presents a new dynamic hysteresis model derived from the LuGre model. The inclusion of the hysteresis friction model within the LuGre friction model yields results that approximate physical closer to the reality than the previous models. The new model shows a flat segment at the corners of the pre-sliding displacement behaviour, produced by the non-elastic deformation shown in Young's modulus.

Benitez S., Acho L. and Guerra R. (2005).
DYNAMIC HYSTERESIS MODEL DERIVATED FROM LuGre MODEL.
In *Proceedings of the Second International Conference on Informatics in Control, Automation and Robotics - Signal Processing, Systems Modeling and Control*, pages 179-183
DOI: 10.5220/0001156201790183

The paper is structured as follows: Section II reviews the LuGre friction model, Section III presents the Hysteresis model, Section IV shows the numeric simulation results, finally, the conclusions are given in Section V.

2 THE LUGRE MODEL

The LuGre model consists of an internal state, which is used to produce friction force, (C. Canudas de Wit, 1995), this model is as follow.

$$\dot{z} = v - \frac{|v|}{g(v)} z \tag{1}$$

$$f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v \tag{2}$$

$$g(v) = (F_C + (F_S - F_C e^{-(v/v_S)^2}))/\sigma_0 \tag{3}$$

where z is the state, *v* is the relative velocity between surfaces, *g(v)* a function that models the constant velocity behaviour, σ_0 an equivalent stiffness for the position-force relationship at velocity reversal, σ_1 the micro-viscous friction coefficient, and σ_2 the viscous friction coefficient. F_C and F_S are the Coulomb friction level and the level of stiction, respectively.
This model can capture most of the known friction behaviours like pre-sliding displacement, friction lag, varying breakaway force and stick-slip motion, and limit cycles, (C. Canudas de Wit, 1995).

Table 1: Parameters values used in all simulations.

Parameters	Values	Unit
$\sigma_o s$	10^5	[N/m]
σ_1	$\sqrt{10^5}$	[N–S/m]
σ_2	0.4	[N–S/m]
F_C	1	[N]
F_S	1.5	[N]
v_S	0.001	[m/s]

2.1 Pre-sliding effect

If a force is applied to a pair of surfaces in contact there will be a displacement. A simulation was performed in (C. Canudas de Wit, 1995), with an applied force slowly ramped up to 1.425 N which is 95% of F_s. The force was then kept constant for a while and later ramped down to -1.425 N, where it was kept constant and then ramped up to 1.425 N again. The results of the simulation are shown in (Fig. 1), this behaviour agrees qualitatively with the experiment results in (J. Courtney-Pratt, 1957).

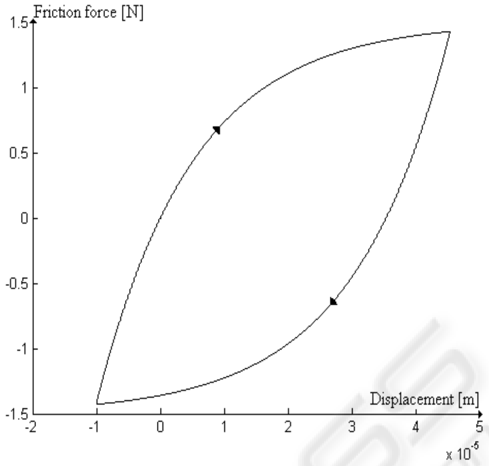

Figure 1: Pre-sliding displacement.

2.2 Friction Lag

Hysteresis behaviour in the relationship between friction and velocity was shown by Hess and Soom (D.P. Hess, 1990) while they were studying the dynamic behaviour of friction when velocity is varied during unidirectional motion. While friction force is lower for decreasing velocities than for increasing velocities, the hysteresis loop becomes wider at higher rates of velocity change. The LuGre model can capture this effect as is shown in (Fig. 2).

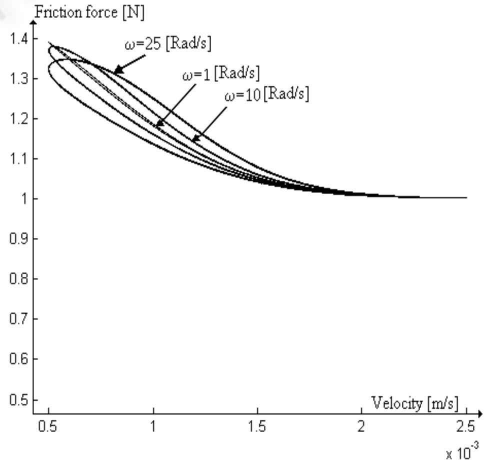

Figure 2: Friction Lag

2.3 Breakaway Force

The breakaway force can be investigated through experiments with stick-slip motion. In (C. Canudas de Wit, 1995) simulations were performed using the dynamic model, where a force was applied to a unit mass and ramped up at different rates, and the friction force when the mass started to slide was determined. The Breakaway force was therefore determined at the time where a sharp increase in the velocity could be observed, (Fig. 3) shows the force at breakaway as a function of the rate of increase of the applied force.

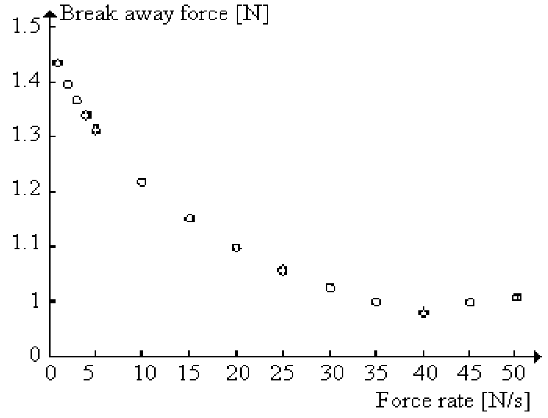

Figure 3: Breakaway Force

2.4 Stick-Slip Motion

Stick-slip motion is a typical behaviour for systems with friction. It is caused by the fact that friction is larger at rest than during motion. In (C. Canudas de Wit, 1995) the author used in his experiments a unit mass attached to a spring with stiffness $k = 2N/m$, and a constant velocity $dy/dt = 0.1m/s$, experimental results are shown in (Fig. 4).

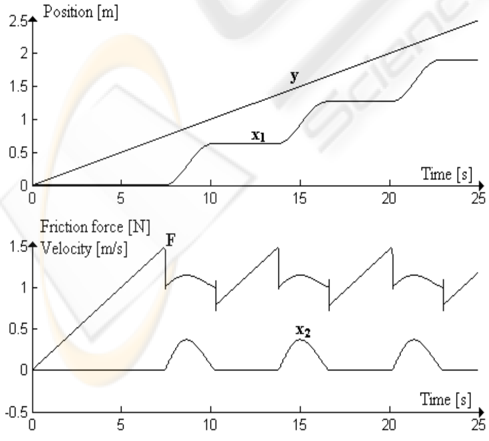

Figure 4: Stick-Slip motion

2.5 Limit Cycle Cause by Friction

Limit cycles are another phenomenon produce by friction, this behaviour is shown in (C. Canudas de Wit, 1995) applying a friction force to a unit mass

$$m\frac{d^2x}{dt^2} = u - F \tag{4}$$

with F as a friction force given by (Eq. 2), and u as control force given by a PID controller

$$u = -k_v v - K_p(x - xd) - K_i \int (x - xd) \tag{5}$$

simulation results are shown in (Fig. 5)

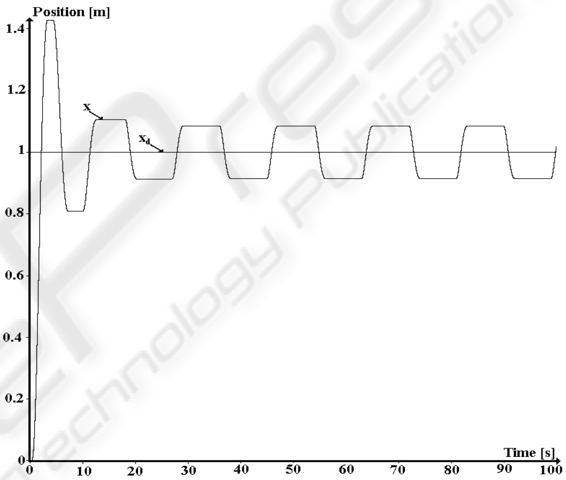

Figure 5: Limit cycle

In (C. Canudas de Wit, 1995) it has been proven that the LuGre model can capture almost all the effects caused by friction, but still some phenomena are missing for instance, what happens when a bristle is out of the elastic region but not yet in the breakaway region, i.e. when it is in the non-elastic deformation region, such kind of phenomena are not captured by LuGre model, here the motivation is to improve this dynamic friction model in order to be able to capture more phenomena produced by friction.

3 HYSTERESIS MODEL

Some papers have been published improving the pre-sliding behaviour adding to the friction equation (Eq. 1) a static hysteresis friction force, see (J. Swevers, 2000) (C. Ganseman, 1997), which is a static function, these improvements can reproduce all the effects shown by the LuGre model.
But neither the LuGre model nor its improvement

(J. Swevers, 2000) can reproduce the pre-sliding displacement in the non-elastic deformation region shown on the stress-strain curve of Young's modulus (F. W. Sears, 1957) (Fig. 6).

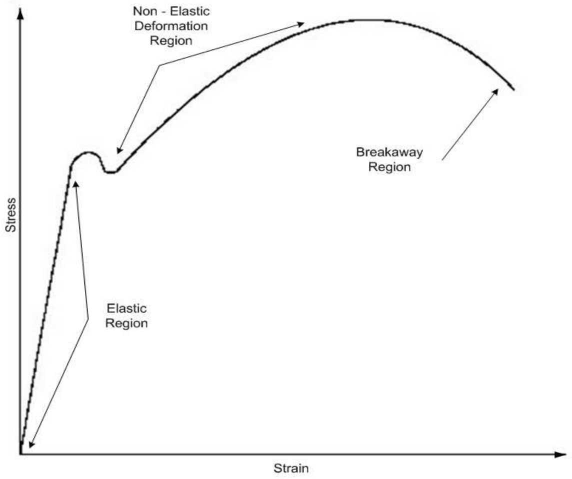

Figure 6: Stress-Strain Curve.

The dynamic hysteresis friction model proposed is derived from the forward and backward displacement of the steady state solution of the LuGre friction model. The steady-state solution is (C. Canudas de Wit, 1995).

$$z_{ss} = \frac{v}{|v|}g(v) = g(v)sgn(v) \qquad (6)$$

If we add a displacement a, we get

$$z_{ss} = \frac{v \pm a}{|v \pm a|}g(v) = g(v)sgn(v \pm a) \qquad (7)$$

After this displacement, represented by the parameter a, it is possible to obtain the behaviour shown in (Fig. 7)

$$\dot{z} = \begin{cases} (v + a) - \frac{|v+a|}{g(v)}z & \dot{v} > 0 \\ (v - a) - \frac{|v-a|}{g(v)}z & \dot{v} < 0 \end{cases} \qquad (8)$$

with a as the displacement factor that can be a constant or could be a function of the state variable z, where a, in some way has a relation with the non-elastic deformation width (non-elastic deformation region Fig. 6), this region is located between the elasticity and breakaway regions of the stress-strain curve (Fig. 6), with this forward and backward displacement it is possible to see the deformation segment of the stress-strain curve, and reproduce all the friction effects reproduced by the LuGre model.
This improvement comes as it was mentioned, from the steady state solution of (Eq. 1)

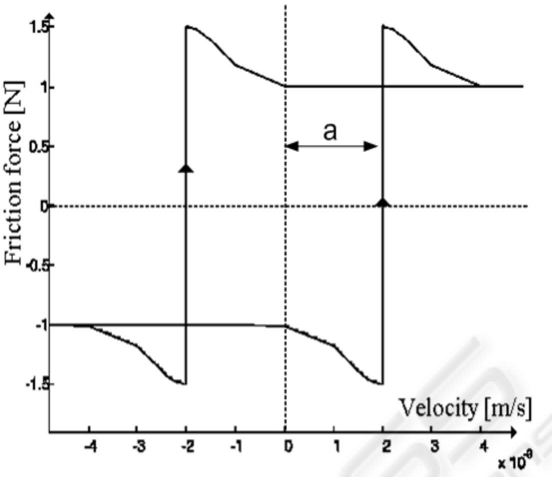

Figure 7: Hysteresis Behavior.

4 NUMERICAL SIMULATION

In order to show the validity of this model, simulations were run on Matlab version 6.0 and the results are shown in the following figures. The same parameters used in (C. Canudas de Wit, 1995) were used in these simulations, where it was found that proposed model can capture the same phenomena that LuGre, such as Frictional lag, varying Breakaway force, Stick-slip motion, and limit cycles, but a new phenomenon was found in the pre-sliding regime, which is assumed to be the non-elastic deformation of the bristles while a force is applied before entering the rupture zone where the displacement starts.
Figure (8) shows the pre-sliding displacement for the model considering that a is a constant value of 0.000002

Figure (9) shows the friction force in the pre-sliding regime behaviour plotted against displacement, for the model considering that a is a minimum function between a constant value of 2×10^{-6} and the norm of the internal state z; i.e., $a = \min\{2 \times 10^{-6}, \|z\|\}$.

It is possible to see a high correlation between (Fig. 9) and (J. Swevers, 2000, Fig. 10), which is an experimental result of displacement-torque curve from the first axis of the KUKA IR 361 robot, the non-elastic deformation region can be seen in both figures.
Figure (10) shows the friction force in the pre-sliding regime behaviour plotted against displacement, for the model considering that a is a maximum function between a constant value of 2×10^{-6} and the norm of the internal state z;i.e., $a = max\{2 \times 10^{-6}, \|z\|\}$.

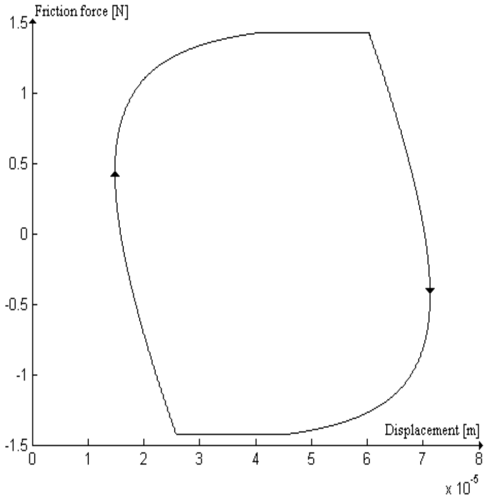

Figure 8: Pre-Sliding displacement.

5 CONCLUSION

A dynamic hysteresis friction model was presented which is valid in both the sliding and pre-sliding regimes. The system is able to accurately capture friction characteristics, experimentally obtained such as: Stribeck effect during sliding, hysteresis behaviour in pre-sliding, frictional lag, varying breakaway force, and stick-slip behaviour, as well as limit cycles. It is concluded that the developed dynamic hysteresis friction model can capture any friction behaviour described in the literature or experimentally observed, but the main improvement with respect to the LuGre model is that the new model can even capture non-elastic deformation, a phenomenon not captured by any dynamic friction model to our knowledge. It is for these reasons that we consider that this model will be of great use, particularly in mechanical applications such as system modeling and control by friction compensation; all though, future work is needed to achieve implementation in the afore mentioned areas.

REFERENCES

C. Canudas de Wit, H. Olsson, K. A. P. L. (1995). A new model for control of system with friction. *IEEE Trans. on Automatic Control*, (3):419–425.

C. Ganseman, J. Swevers, T. P. F. A. B. (1997). An integrated friction model with improved presliding behavior. In *Proc. Symp. Robot Control*, pages 221–226, Nates, France.

Dahl, P. (1968). A solid friction model. Technical Report TOR-0158(3107-18)-1,1968, Aerospace Corp.

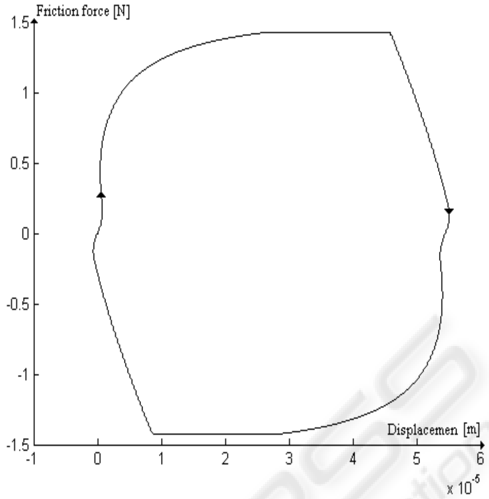

Figure 9: Pre-Sliding displacement.

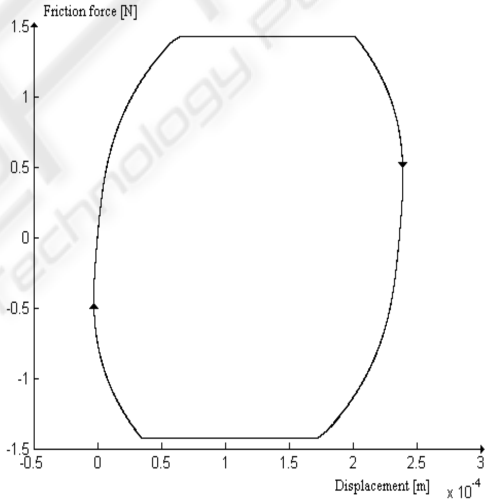

Figure 10: Pre-sliding displacement.

D.P. Hess, A. S. (1990). Friction at a lubricated line contact operating at oscillating sliding velocities. *J. Tribology*, pages 147–152.

F. W. Sears, M. W. Zemansky, H. D. Y. R. A. F. (1957). *Fisica Universitaria*. Addison Wesley Longman, Mexico, 10th edition.

J. Courtney-Pratt, E. E. (1957). The effect of a tangential force on the contact of metallic bodies. In *Proc. Royal Society*, pages 529–550, Nates, France.

J. Swevers, F Al-Bender, C. G. G. T. P. (2000). An integrated friction model structure with improved presliding behavior for accurate friction compensation. *IEEE Trans. on Automatic Control*, (4):675–686.